Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces

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Abstract- F.Smarandache introduced and developed the concept of Neutrosophic set from the fuzzy sets and intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduced and studied about Neutrosophic generalized semi closed sets in Neutrosophic topological spaces and its properties are discussed details.

Index Terms- Neutrosophic semi closed sets, Neutrosophic semi open sets, Neutrosophic generalized semi closed sets, Neutrosophic generalized semi open sets

1. INTRODUCTION


Neutrosophy the degree of indeterminacy, as an independent concept, was introduced by Smarandache [8] in 1998. He also defined the Neutrosophic set on three component Neutrosophic topological spaces (t, i, f) = (Truth, Indeterminacy, Falsehood). The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by Salama [12] et al. R.Dhavaseelan[4], SaiedJafari are introduced Neutrosophic generalized closed sets.K. Bageerathi [10] et al introduced and studied about Neutrosophic semi closed sets in Neutrosophic topological spaces. In this paper we introduced and studied about Neutrosophic generalized semi closed sets in Neutrosophic topological spaces and its properties are discussed details.

2. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operation.

Definition 2.1 [9] 
Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form 

\[ A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \]

where \( \mu_A(x) \), \( \sigma_A(x) \) and \( \gamma_A(x) \) which represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element \( x \in X \) to the set \( A \).

Remark 2.2 [9] 
A Neutrosophic set \( A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \) can be identified to an ordered triple \( (\mu_A, \sigma_A, \gamma_A) \) in \( ]0,1[ \) on X.

Remark 2.3 [9] 
we shall use the symbol \( A = (x, \mu_A, \sigma_A, \gamma_A) \) for the Neutrosophic set \( A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \).

Example 2.4 [9] 
Every Intuitionistic fuzzy set \( A \) is a non-empty set in X is obviously on Neutrosophic set having the form \( A = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \). Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set \( 0_N \) and \( 1_N \) in X as follows:

\( 0_N \) may be defined as:

\[ (0_1) 0_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( (0_2) 0_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( (0_3) 0_N = \{ (x, \sigma_A(x)) : x \in X \} \]

\( (0_4) 0_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( 1_N \) may be defined as:

\[ (1_1) 1_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( (1_2) 1_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( (1_3) 1_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]

\( (1_4) 1_N = \{ (x, \mu_A(x), \sigma_A(x)) : x \in X \} \]
**Definition 2.5** [9]
Let \( A = \{ \mu_A, \sigma_A, \gamma_A \} \) be a Neutrosophic set on \( X \), then the complement of the set \( A \)
\( \{ C(A) \) for short\] defined as
\[
\text{C}(A)=\{(x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)) : x \in X\}.
\]

**Definition 2.6** [9]
Let \( x \) be a non-empty set, and Neutrosophic sets \( A \) and \( B \) in the form
\[
A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \text{ and } B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X \}.
\]
Then we consider definition for subsets \( (A \subseteq B) \).
\[
A \subseteq B \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x) \text{ for all } x \in X.
\]

**Proposition 2.7** [9]
For any Neutrosophic set \( A \), then the following condition are holds :
1. \( 0_T \subseteq A, \ 0_N \subseteq 0_T \)
2. \( 1_T \subseteq A, \ 1_N \subseteq 1_T \)

**Definition 2.8** [9]
Let \( X \) be a non-empty set, and
\[
A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \text{ and } B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X \}.
\]
Then we consider definition for subsets \( (A \subseteq B) \).
\[
A \subseteq B \equiv \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x) \text{ for all } x \in X.
\]

**Proposition 2.10** [9]
For all \( A \) and \( B \) are two Neutrosophic sets then the following condition are true :   
1. \( C(A \cap B) = C(A) \cup C(B) \)
2. \( C(A \cup B) = C(A) \cap C(B) \).

**Definition 11** [11,12]
A Neutrosophic topology is a non-empty set \( X \) is a family of Neutrosophic subsets in \( X \) satisfying the following axioms :   
(i) \( 0_T \subseteq A_T \subseteq X \)
(ii) \( G_1 \cap G_2 \subseteq A_T \) for any \( G_1, G_2 \subseteq X \)
(iii) \( U \subseteq A_T \) for every \( \{ G_i \in E \} \subseteq X \)

The element Neutrosophic topological spaces is called Neutrosophic open sets.

A Neutrosophic set \( F \) is closed if and only if \( C(F) \) is Neutrosophic open.

**Example 2.14** [11,12]
Let \( X = \{ x \} \) and
\[
A_1 = \{( x, 0.6, 0.6, 0.5) : x \in X \} \text{ and } A_2 = \{( x, 0.5, 0.7, 0.9) : x \in X \}.
\]
Then the family \( \tau_N = \{ 0_N, A_1, A_2, A_3, 1_N \} \) is called a Neutrosophic topological space on \( X \).

**Definition 2.15** [11,12]
Let \( X, \tau_N \) be Neutrosophic topological spaces and
\[
A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \}
\]
be a Neutrosophic set in \( X \).

Then the Neutrosophic closure and Neutrosophic interior of \( A \) are defined by
\[
\text{Neu-CI}(A) = \bigcup \{ K : K \text{ is a Neutrosophic closed set in } X \text{ and } A \subseteq K \}.
\]

**Definition 2.16** [11,12]
(i) \( A \) is Neutrosophic open set if and only if \( A = \text{Neu-Int}(A) \).
(ii) \( A \) is Neutrosophic closed set if and only if \( A = \text{Neu-CI}(A) \).

**Proposition 2.17** [11,12]
For any Neutrosophic set \( A \in X, \tau_N \) we have
(i) \( \text{Neu-CI}(C(A)) = C(\text{Neu-Int}(A)) \),
(ii) \( \text{Neu-Int}(C(A)) = C(\text{Neu-CI}(A)) \).

**Proposition 2.18** [11,12]
Let \( X, \tau_N \) be a Neutrosophic topological spaces and \( A, B \) be two Neutrosophic sets in \( X \). Then the following properties are holds :
(i) \( \text{Neu-Int}(A) \subseteq A \).
(ii) \( A \subseteq \text{Neu-CI}(A) \).
(iii) \( A \subseteq \text{Neu-Int}(A) \subseteq \text{Neu-Int}(B) \).
(iv) \( A \subseteq \text{Neu-CI}(A) \subseteq \text{Neu-CI}(B) \).
(v) \( \text{Neu-Int}(\text{Neu-Int}(A)) = \text{Neu-Int}(A) \).
(vi) \( \text{Neu-CI}(\text{Neu-CI}(A)) = \text{Neu-CI}(A) \).
(vii) \( \text{Neu-Int}(A \cap B) = \text{Neu-Int}(A) \cap \text{Neu-Int}(B) \).
(viii) \( \text{Neu-CI}(A \cup B) = \text{Neu-CI}(A) \cup \text{Neu-CI}(B) \).
(ix) \( \text{Neu-Int}(0_N) = 0_N \).
(xi) \( \text{Neu-CI}(0_N) = 0_N \).
(xii) \( \text{Neu-CI}(1_N) = 1_N \).
(xiii) \( A \subseteq \text{Neu-CI}(A) \subseteq C(A) \).
(xiv) \( \text{Neu-CI}(A \cap B) = \text{Neu-CI}(A) \cap \text{Neu-CI}(B) \).
(xv) \( \text{Neu-Int}(A \cup B) = \text{Neu-Int}(A) \cup \text{Neu-Int}(B) \).

**Definition 2.19** [10]
A subset \( A \) of a Neutrosophic space \( (X, \tau_N) \) is called Neutrosophic semi-open if \( A \subseteq \text{Neu-CI}(\text{Neu-int}(A)) \).

The complement of Neutrosophic semi-open set is called Neutrosophic semi-closed.

**Definition 2.20** [4]
Let \( A \) be a subset of a Neutrosophic space \( (X, \tau_N) \) is called generalized Neutrosophic closed( Neu g-closed) if \( \text{Neu-cl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is Neutrosophic.

The complement of a Neu g-closed set is called the Neu g-open set.
3. NEUTROSOPHIC GENERALIZED SEMI CLOSED SET IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the Neutrosophic generalized semi closed set in Neutrosophic topological spaces

Definition 3.1
Let A be a subset of a Neutrosophic space $(X, \tau_N)$ be called Neutrosophic generalized semi closed(Neu-GS-closed) if Neutrosophic semi-cl$A \subseteq U$, whenever $A \subseteq \mathcal{U}$ and $U$ is Neutrosophic open.

The complement of a Neu-GS-closed set is called the Neu-GS-open set.

Example 3.2
Let $X = \{ a, b \}$ and $A_1 = \{(0.4, 0.6, 0.5), (0.7, 0.3, 0.6)\}$
$A_2 = \{(0.3, 0.7, 0.8), (0.6, 0.4, 0.2)\}$
$A_3 = \{(0.4, 0.7, 0.5), (0.7, 0.4, 0.2)\}$
$A_4 = \{(0.3, 0.6, 0.8), (0.6, 0.3, 0.6)\}$
Then $\tau_N = \{ 0_N, A_1, A_2, A_3, A_4, \mathcal{I}_N \}$ is Neutrosophic topological spaces on $X$. Now, $A_3 = \{(0.5, 0.7, 0.5), (0.9, 0.4, 0.5)\}$ is Neutrosophic generalized semi closed set

Definition 3.3
Let $(X, \tau_N)$ be a Neutrosophic topological spaces. Then for a Neutrosophic subset $A$ of $X$, the Neutrosophic semi-interior of $A$ is the union of all Neutrosophic semi-open sets of $X$ contained in $A$, i.e. $\text{Neu-GS-Int}(A) = \cup \{ G : G$ is a Neu-GS open set in $X$ and $G \subseteq A \}$.

Proposition 3.4
Neutrosophic subsets $A$ and $B$ of a Neutrosophic topological spaces $X$ we have

(i) $\text{Neu-GS-Int}(A \cup B) \subseteq \text{Neu-GS-Int}(A) \cup \text{Neu-GS-Int}(B)$

(ii) $A$ is Neu-GS-open set in $X$ $(\iff)$ $\text{Neu-GS-Int}(A) = A$

(iii) $\text{Neu-GS-Int}(\text{Neu-GS-Int}(A)) = \text{Neu-GS-Int}(A)$

(iv) If $A \subseteq B$ then $\text{Neu-GS-Int}(A) \subseteq \text{Neu-GS-Int}(B)$

Proof:
(i) follows from Definition 3.3.

Let $A$ be Neu-GS-open set in $X$. Then $A \subseteq \text{Neu-GS-Int}(A)$. By using(i) we get $A = \text{Neu-GS-Int}(A)$. Conversely assume that $A = \text{Neu-GS-Int}(A)$. By using Definition 3.3, $A$ is Neu-GS-open set in $X$. Thus (ii) is proved. By using(ii), $\text{Neu-GS-Int}(\text{Neu-GS-Int}(A)) = \text{Neu-GS-Int}(A)$. This proves (iii). Since $A \subseteq B$, by using(i), $\text{Neu-GS-Int}(A) \subseteq A \subseteq B$. That is $\text{Neu-GS-Int}(A) \subseteq B$. By(iii), $\text{Neu-GS-Int}(\text{Neu-GS-Int}(A)) \subseteq \text{Neu-GS-Int}(B)$. Thus $\text{Neu-GS-Int}(A) \subseteq \text{Neu-GS-Int}(B)$. This proves (iv).

Theorem 3.5
Let $(X, \tau_N)$ be a Neutrosophic topological spaces. Then for any Neutrosophic subset $A$ and $B$ of a Neutrosophic topological spaces, we have

(i) $\text{Neu-GS-Int}(A \cap B) = \text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B)$

(ii) $\text{Neu-GS-Int}(A \cup B) \supseteq \text{Neu-GS-Int}(A) \cup \text{Neu-GS-Int}(B)$

Proof: Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by using Proposition 3.4(iv), $\text{Neu-GS-Int}(A \cap B) \subseteq \text{Neu-GS-Int}(A)$ and $\text{Neu-GS-Int}(A \cap B) \subseteq \text{Neu-GS-Int}(B)$. This implies that $\text{Neu-GS-Int}(A \cap B) \subseteq \text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B) \subseteq \text{Neu-GS-Int}(A \cup B)$.

By using Proposition 3.4(i), $\text{Neu-GS-Int}(A) \subseteq A$ and $\text{Neu-GS-Int}(B) \subseteq B$. This implies that $\text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B) \subseteq A \cap B$. Now applying Proposition 3.4(iv), $\text{Neu-GS-Int}(\text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B)) \subseteq \text{Neu-GS-Int}(A \cap B) \subseteq \text{Neu-GS-Int}(A \cup B)$. By Proposition 3.2(iii), $\text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B) \subseteq \text{Neu-GS-Int}(A \cup B)$.

From (1) and (2), $\text{Neu-GS-Int}(A \cup B) = \text{Neu-GS-Int}(A) \cap \text{Neu-GS-Int}(B)$. This implies (i). Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by using Proposition 3.4(iv), $\text{Neu-GS-Int}(A \cap \text{Neu-GS-Int}(A \cup B)) \subseteq \text{Neu-GS-Int}(A \cup B)$.

Then $(X, \tau_N)$ is a Neutrosophic topological spaces. Consider the Neutrosophic sets are

$E = \{(0.71, 0.61, 0.11), (0.71, 0.61, 0.11), (0.91, 0.51, 0.01)\}$ and $F = \{(0.41, 0.61, 0.11), (0.51, 0.71, 0.21), (2.1, 0.71, 0.11)\}$.

Then Neu-GS-Int(E) = D and Neu-GS-Int(F) = D. This implies that Neu-GS-Int(E) $\cup$ Neu-GS-Int(F) = D.

Now, $E \cup F = \{(0.71, 0.61, 0.11), (0.71, 0.71, 0.11), (2.01, 0.71, 0.01)\}$, it follows that Neu-GS-Int(E $\cup$ F) = B. Then Neu-GS-Int(E $\cup$ F) $\subseteq$ Neu-GS-Int(E $\cup$ F).

4. NEUTROSOPHIC GENERALIZED SEMI-CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the concept of Neutrosophic generalized semi closure operators in a Neutrosophic topological spaces.
Definition 4.1
Let \((X, \tau_0)\) be a Neutrosophic topological spaces. Then for a Neutrosophic subset \(A\) of \(X\). The Neutrosophic semi-closure of \(A\) is the intersection of all Neutrosophic generalized semi closed sets of \(X\) contained in \(A\). That is, 
\[\text{Neu-GS-CI}(A) = \bigcap \{K : K \text{ is a Neu-GS-C set in } X \text{ and } K \supseteq A\}.\]

Proposition 4.2
Let \((X, \tau_0)\) be a Neutrosophic topological spaces. Then for any Neutrosophic subsets \(A, B\) of \(X\),

(i) \(C(\text{Neu-GS-Int}(A)) = \text{Neu-GS-CI}(C(A))\),

(ii) \(C(\text{Neu-GS-CI}(A)) = \text{Neu-GS-Int}(C(A))\).

Proof:
By using Definition 3.3, 
\[\text{Neu-GS-Int}(A) = C\{G : G \text{ is a Neu-GS-open set in } X \text{ and } G \subseteq A\}.\]
Taking complement on both sides, 
\[C(\text{Neu-GS-Int}(A)) = C(\bigcup \{G : G \text{ is a Neu-GS-open set in } X \text{ and } G \subseteq A\}) = \bigcap \{C(G) : G \text{ is a Neu-GS-C set in } X \text{ and } C(A) \subseteq C(G)\}.\]
Replacing \(C(G)\) by \(K\), we get 
\[C(\text{Neu-GS-Int}(A)) = \bigcap \{K = \text{a Neu-GS-C set in } X \text{ and } K \supseteq C(A)\}.\]
By Definition 4.1, 
\[\text{Neu-GS-CI}(A) = \text{Neu-GS-CI}(C(A)).\]
This proves (i). By using (i), 
\[C(\text{Neu-GS-CI}(A)) = \text{Neu-GS-CI}(C(C(A))) = \text{Neu-GS-CI}(A).\]
Taking complement on both sides, we get 
\[\text{Neu-GS-Int}(C(A)) = C(\text{Neu-GS-CI}(A)).\]
Hence proved (ii).

Proposition 4.3
Let \((X, \tau_0)\) be a Neutrosophic topological spaces. Then for any Neutrosophic subsets \(A, B\) of a Neutrosophic topological spaces \(X\) we have

(i) \(A \subseteq \text{Neu-GS-CI}(A)\),

(ii) \(A\) is Neu-GS-C set in \(X\) if and only if \(\text{Neu-GS-Cl}(A) = A\),

(iii) \(\text{Neu-GS-Cl}(\text{Neu-GS-Cl}(A)) = \text{Neu-GS-Cl}(A)\),

(iv) If \(A \subseteq B\) then \(\text{Neu-GS-Cl}(A) \subseteq \text{Neu-GS-CI}(B)\).

Proof:
(i) follows from Definition 4.2.

Let \(A\) be Neu-GS-Closed set in \(X\). By using Proposition 4.3(i), \(\text{Neu-GS-CI}(A)\) is Neu-GS-open set in \(X\). By Proposition 4.2(ii), \(\text{Neu-GS-Int}(C(A)) = C(A) \Rightarrow C(\text{Neu-GS-Cl}(A)) = C(A) \Rightarrow \text{Neu-GS-Cl}(A) = A\). Thus proved (ii).

By using (ii), \(\text{Neu-GS-CI}(\text{Neu-GS-Cl}(A)) = \text{Neu-GS-CI}(A)\). This proves (iii).

Since \(A \subseteq B\), \(\text{Neu-GS-CI}(B) \subseteq \text{Neu-GS-CI}(A)\). By using Proposition 3.4, \(\text{Neu-GS-Int}(C(B)) \subseteq \text{Neu-GS-Int}(C(A))\). Taking complement on both sides, \(\text{Neu-GS-Cl}(C(B)) \supseteq \text{Neu-GS-Cl}(C(A))\). By Proposition 4.2(ii), \(\text{Neu-GS-CI}(A) \subseteq \text{Neu-GS-Cl}(B)\). This proves (iv).

Proposition 4.4
Let \(A\) be a Neutrosophic set in a Neutrosophic topological spaces \(X\). Then \(\text{Neu-Int}(A) \subseteq \text{Neu-GS-Int}(A) \subseteq \text{Neu-GS-Cl}(A) \subseteq \text{Neu-Cl}(A)\).

Proof:
It follows from the definitions of corresponding operators.

Proposition 4.5
Let \((X, \tau_0)\) be a Neutrosophic topological spaces. Then for a Neutrosophic subset \(A, B\) of a Neutrosophic topological spaces \(X\), we have

(i) \(\text{Neu-GS-Cl}(A \cup B) = \text{Neu-GS-Cl}(A) \cap \text{Neu-GS-Cl}(B)\)

(ii) \(\text{Neu-GS-Cl}(A \cap B) \subseteq \text{Neu-GS-Cl}(A) \cap \text{Neu-GS-Cl}(B)\).

Proof:
Since \(\text{Neu-GS-Cl}(A \cup B) = \text{Neu-GS-Cl}(\text{Cl}(A \cup B))\),
By using Proposition 4.2(ii), \(\text{Neu-GS-Cl}(A \cup B) = (\text{Neu-GS-Int}(C(A \cup B))) = (\text{Neu-GS-Int}(\text{Cl}(A) \cap \text{Cl}(B)))\).
Again using Proposition 3.5, \(\text{Neu-GS-Cl}(A \cup B) = (\text{Neu-GS-Int}(\text{Cl}(A))) \cup (\text{Neu-GS-Int}(\text{Cl}(B)))\).
By using Proposition 4.2(ii), \(\text{Neu-GS-Cl}(A \cap B) = \text{Neu-GS-Cl}(C(A) \cap C(B))\).
This proves (ii). Since \(A \cap B \subseteq A \text{ and } A \cap B \subseteq B\), by using Proposition 4.3(iv), \(\text{Neu-GS-Cl}(A \cap B) \subseteq \text{Neu-GS-Cl}(A) \text{ and } \text{Neu-GS-Cl}(A \cap B) \subseteq \text{Neu-GS-Cl}(B)\).
This implies that \(\text{Neu-GS-Cl}(A \cap B) = \text{Neu-GS-Cl}(A) \cap \text{Neu-GS-Cl}(B)\).
This proves (ii).

The following example shows that the equality need not be hold in Proposition 4.5(ii).

Example 4.6
Let \(X = \{a, b, c\}\) with \(\tau_0 = \{\emptyset, A_1, A_2, A_3, A_4, I_1\}\) and 
\[C(\tau_0) = \{1_{N_1}, A_0, A_0, A_0, 0_{N_1}\}\] where
\[A_0 = \{(0.0, 5.6, 0.1), (0.6, 0.7, 0.1), (0.9, 0.5, 0.2)\}\]
\[A_1 = \{(0.4, 0.5, 0.2), (0.8, 0.6, 0.3), (0.9, 0.7, 0.3)\}\]
\[A_2 = \{(0.4, 0.5, 0.2), (0.6, 0.6, 0.3), (0.9, 0.5, 0.3)\}\]
\[A_3 = \{(0.5, 0.6, 0.1), (0.8, 0.7, 0.1), (0.9, 0.7, 0.2)\}\]
\[A_4 = \{(0.1, 0.4, 0.5), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9)\}\]
\[A_5 = \{(0.2, 0.5, 0.4), (0.3, 0.4, 0.8), (0.3, 0.3, 0.9)\}\]
\[A_6 = \{(0.2, 0.5, 0.4), (0.3, 0.4, 0.6), (0.3, 0.5, 0.9)\}\]
\[A_7 = \{(0.3, 0.4, 0.6), (0.3, 0.5, 0.9)\}\]
Then \((X, \tau_0)\) is a Neutrosophic topological spaces. Consider the Neutrosophic sets are
\[A_0 = \{(0.1, 0.2, 0.5), (0.2, 0.3, 0.7), (0.3, 0.3, 1)\}\]
and
\[A_0 = \{(0.2, 0.4, 0.8), (0.1, 0.2, 0.8), (0.2, 0.5, 0.9)\}\]
Then Neu-GS-Cl(A_0) = A_0 and Neu-GS-Cl(A_0) \cap Neu-GS-Cl(A_0) = A_0.
This implies that Neu-GS-Cl(A_0) \cap Neu-GS-Cl(A_0) = A_0.
Now, A_0 \cap A_0 = \{(0.1, 0.2, 0.8), (0.1, 0.2, 0.8)\}
This follows that Neu-GS-Cl(A_0) \cap Neu-GS-Cl(A_0) = A_0.
Then Neu-GS-Cl(A_0) \cap Neu-GS-Cl(A_0).
By Proposition 4.3(i), \( A \subseteq \text{Neu-GS-Cl}(A) \) -----(1). Again using Proposition 3.4(i), Neu-GS-Int(A) \( \subseteq A \). Then Neu-GS-Cl(Neu-GS-Int(A))\( \subseteq \text{Neu-GS-Cl}(A) \) ----- (2).

By (1) \& (2) we have, \( A \cup \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \subseteq \text{Neu-GS-Cl}(A) \). This proves (i).

By Proposition 3.4(i), Neu-GS-Int(A) \( \subseteq A \) -----(3). Again using proposition 4.3(i), A \( \subseteq \text{Neu-GS-Cl}(A) \). Then Neu-GS-Int(A)\( \subseteq \text{Neu-GS-Int}(\text{Neu-GS-Cl}(A)) \) ----- (4). From (3) \& (4), we have Neu-GS-Int(A) \( \subseteq A \cap \text{Neu-GS-Int}(\text{Neu-GS-Cl}(A)) \). This proves (ii).

By Proposition 4.4, Neu-GS-Cl(A) \( \subseteq \text{Neu-Cl}(A) \). We get Neu-Int(Neu-GS-Cl(A)) \( \subseteq \text{Neu-Int}(\text{Neu-Cl}(A)) \). Hence (iii).

(iii) By (i), Neu-GS-Cl(A) \( \supseteq \) \( A \cup \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \). We have Neu-Int(Neu-GS-Cl(A)) \( \supseteq \text{Neu-Int}(A \cup \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)))) \). Since Neu-Int(A \cup B)\( \supseteq \text{Neu-Int}(A) \cup \text{Neu-Int}(B) \), Neu-Int(Neu-GS-Cl(A)) \( \supseteq \text{Neu-Int}(A) \cup \text{Neu-Int}(\text{Neu-GS-Int}(A)) \). Hence (iv).

**Theorem 4.8**

Let \( (X, \tau) \) be a Neutrosophic topological spaces. Then for a Neutrosophic subset A and B of X we have,

(i) Neu-GS-Cl(A) \( \supseteq \text{A} \cup \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \).

(ii) Neu-GS-Int(A) \( \subseteq A \cap \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \).

(iii) Neu-Int(Neu-GS-Cl(A)) \( \subseteq \text{Neu-Int}(\text{Neu-Cl}(A)) \).

(iv) Neu-Int(Neu-GS-Cl(A)) \( \supseteq \text{Neu-Int}(\text{Neu-GS-Cl}(\text{Neu-GS-Int}(A))) \).

**Proof:**

By Proposition 4.3(i), A \( \subseteq \text{Neu-GS-Cl}(A) \) -----(1). Again using Proposition 3.4(i), Neu-GS-Int(A) \( \subseteq A \). Then Neu-GS-Cl(Neu-GS-Int(A)) \( \subseteq \text{Neu-GS-Cl}(A) \) ---- (2). By (1) \& (2) we have, \( A \cup \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \) \( \subseteq \text{Neu-GS-Cl}(A) \). This proves (i). By Proposition 3.4(i), Neu-GS-Cl(A) \( \subseteq A \) -----(1). Again using proposition 4.3(i), A \( \subseteq \text{Neu-GS-Cl}(A) \). Then Neu-GS-Int(A) \( \subseteq \text{Neu-GS-Cl}(A) \). This proves (ii). By Proposition 4.4, Neu-GS-Cl(A) \( \subsetneq \text{Neu-Cl}(A) \). We get Neu-Int(Neu-GS-Cl(A)) \( \subsetneq \text{Neu-Int}(\text{Neu-Cl}(A)) \). Hence (iii). By (i), Neu-GS-Cl(A) \( \supseteq \text{Neu-GS-Cl}(\text{Neu-GS-Int}(A)) \). We have Neu-Int(Neu-GS-Cl(A)) \( \supseteq \text{Neu-Int}(\text{Neu-GS-Int}(A)) \). Since Neu-Int(A \cup B)\( \supseteq \text{Neu-Int}(A) \cup \text{Neu-Int}(B) \), Neu-Int(Neu-GS-Cl(A)) \( \supseteq \text{Neu-Int}(\text{Neu-GS-Int}(A)) \). Hence (iv).

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