Solution of Population Growth and Decay Problems by Using Mohand Transform

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Abstract: The population growth and decay problems generally appear in the field of zoology, chemistry, social science, biology, physics etc. In the present paper, we applied Mohand transform for solving population growth and decay problems and in application section, some numerical applications are given to explain the usefulness of Mohand transform for solving population growth and decay problems.

Keywords: Mohand transform, Inverse Mohand transform, Population growth problem, Decay problem, Half-life.

1. INTRODUCTION
Mathematically the population growth (growth of a country, or bacteria or a plant, or a cell, or an organ, or a species) is governed by the first order linear ordinary differential equation [1-10]
\[
\frac{dN}{dt} = kN \quad \text{....} \quad \text{....} \quad \text{....} \quad (1)
\]
with initial condition as
\[
N(t_0) = N_0 \quad \text{....} \quad \text{....} \quad (2)
\]
where \(k\) is a positive real number, \(N\) and \(N_0\) are the amount of populations at time \(t\) and initial time \(t_0\).
Equation (1) is called the Malthusian law of population growth.

On the other hand the decay problem of the material (radioactive substance) is mathematically defined by the first order linear ordinary differential equation [7, 9-10]
\[
\frac{dN}{dt} = -kN \quad \text{....} \quad \text{....} \quad \text{....} \quad (3)
\]
with initial condition as
\[
N(t_0) = N_0 \quad \text{....} \quad \text{....} \quad (4)
\]
where \(N\) is the amount of the material at time \(t\), \(k\) is a positive real number and \(N_0\) is the amount of the material at initial time \(t_0\).

Since the mass of the substance decreases with time and so the time derivative of \(N\) denoted by \(\frac{dN}{dt}\) must be negative. This is the reason behind taking the negative sign in the R.H.S. of equation (3).
The Mohand transform of the function \(F(t)\) is defined as [11]:
\[
M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt
\]
\[= R(v), t \geq 0, k_1 \leq v \leq k_2\]
where \(M\) is Mohand transform operator.
The sufficient conditions for the existence of Mohand transform of the function \(F(t)\) for \(t \geq 0\) are
a) \(F(t)\) must be piecewise continuous.
b) \(F(t)\) must be of exponential order.

Kumar et al. [12] used Mohand transform for solving linear Volterra integro-
Aggarwal et al. [13] solved population growth and decay problems using Laplace transform. Aggarwal et al. [14] applied Kamal transform to solve the population growth and decay problems. Application of Elzaki transform for solving population growth and decay problems was given by Aggarwal et al. [15]. Aggarwal et al. [16] used Mahgoub transform for solving population growth and decay problems. Aggarwal et al. [17] applied Aboodh transform for solving the population growth and decay problems. Aggarwal et al. [18] gave the solution of linear Volterra integral equations of second kind using Mohand transform. Mohand transform of Bessel’s functions was defined by Aggarwal et al. [19].

The aim of the present work is to finding the solution of population growth and decay problems by applying Mohand transform without any large calculation work.

### 2. LINEARITY PROPERTY OF MOHAND TRANSFORM

If \( \{F(t)\} = H(v) \) and \( \{G(t)\} = I(v) \) then \( \{aF(t) + bG(t)\} = a\{F(t)\} + b\{G(t)\} \)

\[ \Rightarrow \{aF(t) + bG(t)\} = aH(v) + bI(v), \]

where \( a, b \) are arbitrary constants.

### 3. MOHAND TRANSFORM OF SOME ELEMENTARY FUNCTIONS [11-12]

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( F(t) )</th>
<th>( M{F(t)} = R(v) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( v )</td>
</tr>
<tr>
<td>2.</td>
<td>( t )</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>( t^2 )</td>
<td>( \frac{2!}{v} )</td>
</tr>
<tr>
<td>4.</td>
<td>( t^n, n \in \mathbb{N} )</td>
<td>( \frac{n!}{v^{n-1}} )</td>
</tr>
<tr>
<td>5.</td>
<td>( t^n, n &gt; -1 )</td>
<td>( \frac{\Gamma(n+1)}{v^{n-1}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^{at} )</td>
<td>( \frac{v^2}{v-a} )</td>
</tr>
</tbody>
</table>

### 4. INVERSE MOHAND TRANSFORM

If \( M\{F(t)\} = R(v) \) then inverse Mohand transform of \( R(v) \) is given by \( F(t) = M^{-1}\{R(v)\} \)

where \( M^{-1} \) is the inverse Mohand transform operator.

### 5. LINEARITY PROPERTY OF INVERSE MOHAND TRANSFORM

If \( M^{-1}\{H(v)\} = F(t) \) and \( M^{-1}\{I(v)\} = G(t) \) then \( M^{-1}\{aH(v) + bI(v)\} = aM^{-1}\{H(v)\} + bM^{-1}\{I(v)\} \)

\[ \Rightarrow M^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t), \]

where \( a, b \) are arbitrary constants.

### 6. INVERSE MOHAND TRANSFORM OF SOME ELEMENTARY FUNCTIONS

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( R(v) )</th>
<th>( F(t) = M^{-1}{R(v)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( v )</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>( t )</td>
<td>( \frac{1}{t} )</td>
</tr>
<tr>
<td>3.</td>
<td>( t^2 )</td>
<td>( \frac{1}{2v} )</td>
</tr>
<tr>
<td>4.</td>
<td>( \frac{1}{v^{n-1}} )</td>
<td>( \frac{t^n}{n!}, n \in \mathbb{N} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \frac{1}{v^{n-1}} )</td>
<td>( \frac{t^n}{\Gamma(n+1)}, n &gt; -1 )</td>
</tr>
<tr>
<td>6.</td>
<td>( \frac{v^2}{v-a} )</td>
<td>( e^{at} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \frac{v^2}{(v^2 + a^2)} )</td>
<td>( \frac{\sin at}{a} )</td>
</tr>
</tbody>
</table>
8. \[ \frac{v^3}{(v^2 + a^2)} \quad \text{cosat} \]

9. \[ \frac{v^2}{(v^2 - a^2)} \quad \text{sinhat} \]

10. \[ \frac{v^3}{(v^2 - a^2)} \quad \text{coshat} \]

7. MOHAND TRANSFORM OF THE DERIVATIVES OF THE FUNCTION \( F(t) \) [11-12]

If \( M\{F(t)\} = R(v) \) then

a) \( M\{F'(t)\} = vR(v) - v^2 F(0) \)

b) \( M\{F''(t)\} = v^2 R(v) - v^3 F(0) - v^2 F'(0) \)

c) \( M\{F^{(n)}(t)\} = v^n R(v) - v^{n+1} F(0) - v^n F'(0) - \cdots - v^2 F^{(n-1)}(0) \)

8. MOHAND TRANSFORM METHOD FOR POPULATION GROWTH PROBLEM

In this section, we explain the procedure of solving population growth problem which is mathematically given by (1) and (2) by applying Mohand transform.

Applying the Mohand transform on both sides of (1), we have

\[ M\left\{ \frac{dN}{dt} \right\} = kM\{N(t)\} \]

(5)

Now applying the property, Mohand transform of derivative of function, on (5), we have

\[ vM\{N(t)\} - v^2 N(0) = kM\{N(t)\} \]

Using (2) in (6) and on simplification, we have

\( (v - k) M\{N(t)\} = v^2 N_0 \)

\[ \Rightarrow M\{N(t)\} = \frac{v^2 N_0}{(v - k)} \]

(7)

Operating inverse Mohand transform on both sides of (7), we get the required amount of the population at time \( t \).

\[ N(t) = N_0 M^{-1} \left\{ \frac{v^2}{(v - k)} \right\} \]

\[ \Rightarrow N(t) = N_0 e^{kt} \]

(8)

9. MOHAND TRANSFORM METHOD FOR DECAY PROBLEM

In this section, we explain the procedure of solving decay problem which is given by (3) and (4) by applying Mohand transform.

Applying the Mohand transform on both sides of (3), we have

\[ M\left\{ \frac{dN}{dt} \right\} = -kM\{N(t)\} \]

(9)

Now applying the property, Mohand transform of derivative of function, on (9), we have

\[ vM\{N(t)\} - v^2 N(0) = -kM\{N(t)\} \]

Using (4) in (10) and on simplification, we have

\( (v + k) M\{N(t)\} = v^2 N_0 \)

\[ \Rightarrow M\{N(t)\} = \frac{v^2 N_0}{(v + k)} \]

(11)

Operating inverse Mohand transform on both sides of (11), we get the required amount of substance at time \( t \).

\[ N(t) = N_0 M^{-1} \left\{ \frac{v^2}{(v + k)} \right\} \]

\[ \Rightarrow N(t) = N_0 e^{-kt} \]

(12)

10. APPLICATIONS

In this section, some numerical applications are given to explain the usefulness of Mohand transform for solving population growth and decay problems.

**Application: 10.1** The population of Australia grows at a rate proportional to the number of people presently living in Australia. If after five years, the population has doubled, and after ten years the population is 20,000, then estimate the number of people initially living in Australia.

Mathematically the above problem can be written as:

\[ \frac{dN(t)}{dt} = kN(t) \]

(13)

where \( N \) denote the number of people living in the country at any time \( t \) and \( k \) is the constant
of proportionality. Consider \( N_0 \) is the number of people initially living in Australia at \( t = 0 \).

Applying the Mohand transform on both sides of (13), we have

\[
M \left\{ \frac{dN(t)}{dt} \right\} = kM\{N(t)\} \quad \ldots \ldots \ldots \ldots (14)
\]

Now applying the property, Mohand transform of derivative of function, on (14), we have

\[
vM\{N(t)\} - v^2N(0) = kM\{N(t)\} \quad \ldots \ldots \ldots \ldots (15)
\]

Since at \( t = 0, N = N_0 \), so using this in (15), we have

\[
(v - k) M\{N(t)\} = v^2N_0
\]

\[
\Rightarrow M\{N(t)\} = \frac{v^2N_0}{(v - k)} \quad \ldots \ldots \ldots \ldots (16)
\]

Operating inverse Mohand transform on both sides of (16), we have

\[
N(t) = N_0 M^{-1} \left\{ \frac{v^2}{(v - k)} \right\} = N_0 e^{kt} \quad \ldots \ldots (17)
\]

Now at \( t = 5, N = 2N_0 \), so using this in (17), we have

\[
2N_0 = N_0 e^{5k}
\]

\[
\Rightarrow e^{5k} = 2
\]

\[
\Rightarrow k = \frac{1}{5} \log_e 2 = 0.1386 \quad \ldots \ldots \ldots (18)
\]

Now using the condition at \( t = 10, N = 20,000 \), in (17), we have

\[
20,000 = N_0 e^{10k} \quad \ldots \ldots \ldots (19)
\]

Now substituting the value of \( k \) from (18) to (19), we get the required number of people initially living in Australia.

\[
20,000 = N_0 e^{10 \times 0.1386}
\]

\[
\Rightarrow 20,000 = 3.9988N_0
\]

\[
\Rightarrow N_0 = 5001.5 \quad \ldots \ldots \ldots \ldots (20)
\]

**Application: 10.2** A radioactive material is known to decay at a rate proportional to the amount present. If initially there is 500 milligrams of the radioactive material present and after five hours it is observed that the radioactive material has lost 25 percent of its original mass, find the half life of the radioactive material.

Mathematically the above problem can be written as:

\[
\frac{dN(t)}{dt} = -kN(t) \quad \ldots \ldots \ldots (21)
\]

where \( N \) denote the amount of radioactive material at time \( t \) and \( k \) is the constant of proportionality. Consider \( N_0 \) is the initial amount of the radioactive material at time \( t = 0 \).

Applying the Mohand transform on both sides of (21), we have

\[
M \left\{ \frac{dN(t)}{dt} \right\} = -kM\{N(t)\} \quad \ldots \ldots \ldots \ldots (22)
\]

Now applying the property, Mohand transform of derivative of function, on (22), we have

\[
vM\{N(t)\} - v^2N(0) = -kM\{N(t)\} \ldots \ldots \ldots \ldots (23)
\]

Since at \( t = 0, N = N_0 = 500 \), so using this in (23), we have

\[
vM\{N(t)\} - 500v^2 = -kM\{N(t)\}
\]

\[
\Rightarrow (v + k) M\{N(t)\} = 500v^2
\]

\[
\Rightarrow M\{N(t)\} = \frac{500v^2}{(v + k)} \quad \ldots \ldots \ldots \ldots (24)
\]

Operating inverse Mohand transform on both sides of (24), we have

\[
N(t) = 500 M^{-1} \left\{ \frac{v^2}{(v + k)} \right\}
\]

\[
\Rightarrow N(t) = 500 e^{-kt} \quad \ldots \ldots \ldots (25)
\]

Now at \( t = 5 \), the radioactive material has lost 25 percent of its original mass 500 mg so \( N = 500 - 125 = 375 \), using this in (25), we have

\[
375 = 500 e^{-5k}
\]

\[
\Rightarrow e^{-5k} = 0.75
\]

\[
\Rightarrow k = -\frac{1}{5} \log_e 0.75 = 0.0575 \quad \ldots \ldots \ldots (26)
\]

We required \( t \) when \( N = \frac{N_0}{2} = \frac{500}{2} = 250 \) so from (25), we have

\[
250 = 500 e^{-kt} \quad \ldots \ldots \ldots \ldots (27)
\]

Now substituting the value of \( k \) from (26) to (27), we get the required half-life of the radioactive material.

\[
250 = 500 e^{-0.0575t}
\]
11. CONCLUSION

In the present paper, we have successfully discussed the Mohand transform for solving the population growth and decay problems. The given numerical applications in application section show that the usefulness of Mohand transform for solving population growth and decay problems. These applications also show that there is no need of very large computational work for solving population growth and decay problems using Mohand transform. In the future, we can easily solve the problems of mechanics and electrical circuit using the present scheme.

REFERENCES


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