Neutrosophic Generalized Regular Contra Continuity in Neutrosophic Topological Spaces

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Abstract- In this paper, the concept of Neutrosophic generalized regular contra continuous mapping are introduced. Furthermore Neutrosophic generalized regular contra irresolute mapping, Strongly Neutrosophic generalized regular contra continuous mapping and Perfectly Neutrosophic generalized regular contra continuous mapping are introduced in Neutrosophic topological spaces.

Index terms- Neutrosophic generalized regular contra continuity; Neutrosophic generalized regular contra irresolute; Strongly Neutrosophic generalized regular contra continuity and Perfectly Neutrosophic generalized regular contra continuity.

1. INTRODUCTION


2. PRELIMINARIES

Definition 2.1 [8,9]
Let X be a non-empty fixed set. A Neutrosophic set A has the form
\[ A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \]
where \(\mu_A(x), \sigma_A(x), \gamma_A(x)\) are topological spaces and \(\mu_A(x)\) is the degree of membership function, \(\sigma_A(x)\) is the degree of indeterminacy and \(\gamma_A(x)\) is the degree of non-membership function respectively of each \(x \in X\) to the set \(A\).

Remark 2.2 [8,9]
A Neutrosophic set \(A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}\) can be identified to an ordered triple \((\mu_A, \sigma_A, \gamma_A)\) in \([0^+, 1^-] \cup \{0\}\) on \(X\).

Example 2.3 [8,9]
Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set \(0_N\) and \(1_N\) in \(X\) as follows:
\[ 0_N = \{(x, 0, 0, 1) : x \in X\} \]
\[ 1_N = \{(x, 1, 0, 0) : x \in X\} \]

Definition 2.4 [4]
Let \(A = \{(x, \mu_A, \sigma_A, \gamma_A)\}\) be Neutrosophic set on \(X\), then Complement of set \(A\) i.e., \(A^C\) is defined as \(A^C = \{(x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x)) : x \in X\}\).

Definition 2.5 [8,9]
Let \(X\) be a non-empty set and \(A\) and \(B\) are Neutrosophic sets of the form \(A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}\) and \(B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\}\) then we consider the definition of subset \(A \subseteq B\) as defined as \(A \subseteq B \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x),\) for all \(x \in X\).

Theorem 2.6 [4]
For any Neutrosophic set \(A\) the following condition holds
(i) \(0_N \subseteq A, 0_N \subseteq 0_N\),
(ii) \(A \equiv 1_N, 1_N \equiv 1_N\).

Definition 2.7 [8,9]
Let \(X\) be a non-empty set and \(A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x))\}\) and
B = \{x, \mu_B(x), \sigma_B(x), \gamma_B(x)\} are Neutrosophic sets then \(A \cap B\) is defined as
\(A \cap B = \{x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\}\)
then \(A \cup B\) is defined as
\(A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\}\).

**Definition 2.8** [8,9]
A Neutrosophic topology is a non-empty set \(X\) is an family \(\tau_X\) of Neutrosophic subsets in \(X\) satisfying the axioms:
(i) \(0_X, 1_X \in \tau_X\)
(ii) \(G_i \cap G_j \in \tau_X\) for any \(G_i, G_j \in \tau_X\)
(iii) \(\cup G_i \in \tau_X\) for every \(\{G_i\}_{i \in J} \subseteq \tau_X\)

The element in Neutrosophic topological space \((X, \tau_X)\) are called Neutrosophic open sets.
A Neutrosophic set \(F\) is closed if and only if \((F)^C\) is Neutrosophic open.

**Definition 2.9** [8,9]
Let \((X, \tau_X)\) Neutrosophic topological space and \(A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x) : x \in X\}\) be Neutrosophic set in \(X\). Then the Neutrosophic closure and Neutrosophic interior are defined as:
\(Ncl(A) = \{K : K is Neutrosophic closed set in X and A \subseteq K\}\)
\(Nint(A) = \{G : G is Neutrosophic open set in X and G \subseteq A\}\).

**Definition 2.10** [4]
A is Neutrosophic open set if and only if \(A = Nint(A)\).
\(A\) is Neutrosophic closed set if and only if \(A = Ncl(A)\).

**Theorem 2.11** [4]
Let \((X, \tau_X)\) Neutrosophic topological spaces and \(A, B\) be two Neutrosophic sets in \(X\). Then the following properties holds:
(i) \(A \subseteq B \Rightarrow Nint(A) \subseteq Nint(B)\),
(ii) \(A \subseteq B \Rightarrow Ncl(A) \subseteq Ncl(B)\),
(iii) \(Nint(Nint(A)) = Nint(A)\),
(iv) \(Ncl(Ncl(A)) = Ncl(A)\),
(v) \(Nint(A \cap B) = Nint(A) \cap Nint(B)\),
(vi) \(Ncl(A \cup B) = Ncl(A) \cup Ncl(B)\),
(vii) \(Nint(0_X) = 0_X\),
(viii) \(Nint(1_X) = 1_X\),
(ix) \(Ncl(0_X) = 0_X\),
(x) \(Ncl(1_X) = 1_X\),
(xi) \(Ncl(A \cap B) \subseteq Ncl(A) \cap Ncl(B)\),
(xii) \(Nint(A \cup B) \supseteq Nint(A) \cup Nint(B)\).

**Definition 2.12** [2,4]
A subset \(A\) of Neutrosophic space \((X, \tau_X)\) is called Neutrosophic regular open (in short \(NR\) open) if \(A = \text{Nint}(\text{Ncl}(A))\). The Complement of \(NR\) open set is called \(NR\) closed.

**Definition 2.13** [4]
A subset \(A\) of Neutrosophic space \((X, \tau_X)\) is called Neutrosophic generalized closed (in short \(NG\) closed) if \(\text{Ncl}(A) \subseteq U\), whenever \(A \subseteq U\) and \(U\) is Neutrosophic open. The Complement of a \(NG\) closed set is called \(NG\) open set.

**Definition 2.14** [4]
Let \(A\) be a subset of Neutrosophic space \((X, \tau_X)\) is called Neutrosophic generalized regular closed (\(NGR\) closed) if \(\text{Neutrosophic Regular cl}(A) \subseteq U\) (in short \(NRcl\) closed) whenever \(A \subseteq U\) and \(U\) is Neutrosophic open. The Complement of a \(NGR\) closed set is called \(NGR\) open set.

**Definition 2.15** [5]
Let \((X, T)\) and \((Y, S)\) be any two Neutrosophic topological spaces
(i) A map \(f : (X, T) \rightarrow (Y, S)\) is said to be Neutrosophic continuous if the inverse image of every Neutrosophic closed set in \((Y, S)\) is Neutrosophic closed set in \((X, T)\).

**Definition 2.16** [5]
Let \((X, T)\) and \((Y, S)\) be any two Neutrosophic topological spaces
(i) A map \(f : (X, T) \rightarrow (Y, S)\) is said to be Neutrosophic generalized regular continuous (in short \(NGR\) continuous) if the inverse image of every Neutrosophic closed set in \((Y, S)\) is \(NGR\) closed set in \((X, T)\).

(ii) A map \(f : (X, T) \rightarrow (Y, S)\) is said to be Neutrosophic generalized regular irresolute (in short \(NGR\) irresolute) if the inverse image of every \(NGR\) closed set in \((Y, S)\) is \(NGR\) closed set in \((X, T)\).

(iii) A map \(f : (X, T) \rightarrow (Y, S)\) is said to be Strongly Neutrosophic generalized regular continuous (in short \(NGR\) continuous) if the inverse image of every \(NGR\) open set in \((Y, S)\) is an Neutrosophic open set in \((X, T)\).

**Definition 2.17** [9]
Let \((X, T)\) and \((Y, S)\) be any two neutrosophic topological spaces.
(i) A function \(f : (X, T) \rightarrow (Y, S)\) is called Neutrosophic contra-continuous if the inverse image of every Neutrosophic open set in \((Y, S)\) is a Neutrosophic closed set in \((X, T)\).
Let (X, T ) and (Y, S) be any two Neutrosophic topological spaces.

(i) A function \( f : (X, T ) \rightarrow (Y, S) \) is said to be Neutrosophic regular contra-continuous (in short NR contra-continuous) if the inverse image of every Neutrosophic open set in \((Y, S)\) is a NR closed set in \((X, T)\).

(ii) A function \( f : (X, T ) \rightarrow (Y, S) \) is called a Neutrosophic generalized regular contra-continuous (in short GNR contra-continuous) if \( f^{-1}(B) \) is a GNR closed set in \((X, T)\) for every Neutrosophic open set \(B\) in \((Y, S)\).

(iii) A function \( f : (X, T ) \rightarrow (Y, S) \) is called a Strongly Neutrosophic generalized regular contra-continuous (in short Strongly GNR contra-continuous) if \( f^{-1}(B) \) is a Neutrosophic closed set in \((X, T)\) for every GNR open set \(B\) in \((Y, S)\).

(iv) A function \( f : (X, T ) \rightarrow (Y, S) \) is called a Neutrosophic generalized regular contra irresolute (in short GNR contra irresolute), if \( f^{-1}(B) \) is a GNR closed set in \((X, T)\) for every GNR open set \(B\) in \((Y, S)\).

**Theorem 3.2**

Let \( (X, T) \) and \( (Y, S) \) be any two Neutrosophic topological spaces. If \( f : (X, T) \rightarrow (Y, S) \) is Neutrosophic contra-continuous function. Then it is a GNR contra-continuous mapping.

**Proof:**

Let \( B \) be a Neutrosophic open sets in \((Y, S)\).

Since \( f \) is Neutrosophic contra-continuous function,

By definition 2.17(ii) \( f^{-1}(B) \) is Neutrosophic closed set in \((X, T)\).

We know, Every Neutrosophic closed set is GNR closed set.

Now \( f^{-1}(B) \) is GNR closed set in \((X, T)\).

Therefore By definition 3.1(ii), \( f \) is GNR contra-continuous mapping. Hence proved.

The converse of theorem 3.2 need not be true as shown in example 3.2.1

**Example 3.2.1**

Let \( X = \{a, b\} \) and where \( A = \{0.6, 0.6, 0.6\} \) and \( B = \{0.4, 0.4, 0.4\} \) is Neutrosophic sets. Then the families \( T = \{0_N, 1_N\} \) and \( S = \{0_N, 1_N\} \) are Neutrosophic topologies.

Let \( A = \{0.4, 0.6, 0.5\}, \{0.5, 0.5, 0.5\} \) and \( B = \{0.4, 0.4, 0.4\} \) is Neutrosophic sets. Then the families \( T = \{0_N, 1_N\} \) and \( S = \{0_N, 1_N\} \) are Neutrosophic topologies.

**Theorem 3.3**

Every NR closed sets is GNR closed set. But the converse is not true.

**Proof:**

Let \( A \) be NR closed sets in \( X \) and let \( A \subseteq U \) and \( U \) is Neutrosophic open set in \( X \).

We know that \( A = \text{Ncl}(\text{Nint}(A)) \) (By definition 2.12) \( \Rightarrow \text{Ncl}(\text{Nint}(A)) \subseteq U \)

That is \( \text{NRcl}(A) \subseteq U \)

Therefore we have \( \text{NRcl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is Neutrosophic open set in \( X \).

Therefore by definition 2.14, \( A \) is GNR closed set in \( X \). Hence proved.

The converse of theorem 3.3 is not true is shown in example 3.4

**Example 3.4**

Let \( X = \{a, b\} \) and where \( A_1 = \{0.4, 0.6, 0.5\} \), \( \{0.7, 0.3, 0.6\} \) \( \subseteq \)

\( A_2 = \{0.3, 0.6, 0.8\}, \{0.6, 0.3, 0.6\} \) and \( \tau_N = \{0_N, \}

\( A_3 = \{0.5, 0.7, 0.5\}, \{0.9, 0.4, 0.5\} \) is GNR closed set but not NR closed set.
on X. Define a function $f : (X, T) \rightarrow (Y, S)$ as $f(a) = a$, $f(b) = b$. Then $f$ is a NGR contra-continuous function but $f^{-1}(B)$ is not a NR closed set in $(X, T)$. Hence $f$ not a NGR contra-continuous function.

**Theorem 3.6**
For any two Neutrosophic topological spaces $(X, T)$ and $(Y, S)$. If $f : (X, T) \rightarrow (Y, S)$ is a Strongly NGR contra-continuous function then $f$ is a NGR contra-continuous function.

Proof:
Let $B$ be a Neutrosophic open set in $(Y, S)$. We know, Every Neutrosophic open set is a NGR open set. Now, $B$ is a NGR open set in $(Y, S)$.

Since $f$ is a Strongly NGR contra-continuous function, By definition 3.1(iii), $f^{-1}(B)$ is a Neutrosophic closed set in $(X, T)$.

Since every Neutrosophic closed set is a NGR closed set $f^{-1}(B)$ is a NGR closed set in $(X, T)$.

Hence by definition 3.1(ii), $f$ is a NGR contra-continuous function. Hence proved.

The converse of theorem 3.6 need not be true as shown in example 3.6.1.

**Example 3.6.1**
Let $X = \{a, b\}$ and where $A = \{(0.4, 0.4, 0.4), (0.3, 0.3, 0.3)\}$ and $B = \{(0.2, 0.2, 0.3), (0.8, 0.8, 0.7)\}$ is Neutrosophic sets. Then the families $T = \{0_N, 1_N\}$, $A$ and $S = \{0_S, 1_S\}$ are Neutrosophic topologies on $X$. Define a function $f : (X, T) \rightarrow (Y, S)$ as $f(a) = a$, $f(b) = b$. Then $f$ is a NGR contra-continuous function. Let $C = \{(0.4, 0.4, 0.4), (0.6,0.6,0.6)\}$ be a Neutrosophic open set in $(X, T)$ set, but $f^{-1}(C)$ is not a Neutrosophic closed set in $(X, T)$. Hence $f$ is not a Strongly NGR contra-continuous function.

**Theorem 3.7**
For any two Neutrosophic topological spaces $(X, T)$ and $(Y, S)$. If $f : (X, T) \rightarrow (Y, S)$ is a Strongly NGR contra-continuous function then $f$ is a Neutrosophic contra-continuous function.

Proof:
Let $B$ be a Neutrosophic open set in $(Y, S)$. We know, Every Neutrosophic open set is a NGR open set. Now, $B$ is a NGR open set in $(Y, S)$.

Since $f$ is a Strongly NGR contra-continuous function, By definition 3.1(iii), $f^{-1}(B)$ is a Neutrosophic closed set in $(X, T)$.

Hence by definition 2.17(i), $f$ is a Neutrosophic contra-continuous function. Hence proved.

**Theorem 3.8**
Let $(X, T), (Y, S)$ and $(Z, R)$ be any three Neutrosophic topological spaces. If a function $f : (X, T) \rightarrow (Y, S)$ is strongly NGR continuous function and $g : (Y, S) \rightarrow (Z, R)$ is a NGR contra-continuous function then $g \circ f$ is a Neutrosophic contra-continuous function.

Proof:
Let $B$ be a Neutrosophic open set in $(Z, R)$. Since $g$ is a NGR contra-continuous function, By definition 3.1(ii) , $g^{-1}(B)$ is NGR closed set in $(Y, S)$.

Since $f$ is Strongly NGR continuous function, By definition 2.16(iii) , $f^{-1}(g^{-1}(B))$ is a Neutrosophic closed set in $(X, T)$.

Hence by definition 2.17(i) , $g \circ f$ is a Neutrosophic contra-continuous function. Hence proved.

**Theorem 3.9**
Let $(X, T), (Y, S)$ and $(Z, R)$ be any three Neutrosophic topological spaces. If $f$ is a NGR contra-continuous function and $g$ is a Neutrosophic continuous function, then $g \circ f$ is a NGR contra-continuous function.

Proof:
Let $B$ be a Neutrosophic open set in $(Z, R)$. Since $g$ is a Neutrosophic continuous function, By definition 2.15(i) , $g^{-1}(B)$ is Neutrosophic open set in $(Y, S)$.

Since $f$ is a NGR contra-continuous function, By definition 3.1(ii) , $f^{-1}(g^{-1}(B))$ is a NGR closed set in $(X, T)$.

Hence by definition 4.1(ii) , $g \circ f$ is a NGR contra-continuous function. Hence proved.

**Theorem 3.10**
Let $(X, T), (Y, S)$ and $(Z, R)$ be any three Neutrosophic topological spaces. If $f$ is a NGR contra-continuous function and $g$ is a Neutrosophic contra-continuous function, then $g \circ f$ is a NGR continuous function.

Proof:
Let $B$ be a Neutrosophic open set in $(Z, R)$. Since $g$ is a Neutrosophic contra-continuous function, By definition 2.17(i) , $g^{-1}(B)$ is Neutrosophic closed set in $(Y, S)$.

Since $f$ is a NGR contra-continuous function, By definition 3.1(ii) , $f^{-1}(g^{-1}(B))$ is a NGR open set in $(X, T)$.

Hence by definition 3.1(i) , $g \circ f$ is a NGR continuous function. Hence proved.

**Theorem 3.11**
Let $(X, T), (Y, S)$ and $(Z, R)$ be any three Neutrosophic topological spaces. If $f$ is a NGR contra-irresolute function and $g$ is a NGR contra-continuous function, then $g \circ f$ is a NGR continuous function.
Proof: 
Let $B$ be a Neutrosophic open set in $(Z, R)$. 
Since $g$ is a $NGR$ $contra$-$continuous$ function, 
By definition 3.1(i), $g^{-1}(B)$ is $NGR$ closed set in $(Y, S)$. 
Since $f$ is a $NGR$ $contra$-irresolute function, 
By definition 3.1(iv), $f^{-1}(g^{-1}(B))$ is a $NGR$ open set in $(X, T)$. 
Hence by definition 2.16(i), $g \circ f$ is a $NGR$ continuous function. Hence proved. 

**Theorem 3.12**

Let $(X, T), (Y, S)$ and $(Z, R)$ be any three Neutrosophic topological spaces. If $f$ is a $NGR$ irresolute function and 
g is a $NGR$ $contra$-continuous function, then $g \circ f$ is a $NGR$ $contra$-continuous function. 
Proof: 
Let $B$ be a Neutrosophic open set in $(Z, R)$. 
Since $g$ is a $NGR$ $contra$-$continuous$ function, 
By definition 3.1(ii), $g^{-1}(B)$ is $NGR$ closed set in $(Y, S)$. 
Since $f$ is a $NGR$ irresolute function, 
By definition 2.16(ii), $f^{-1}(g^{-1}(B))$ is a $NGR$ closed set in $(X, T)$. 
Hence by definition 3.1(ii), $g \circ f$ is a $NGR$ $contra$-continuous function. Hence proved. 

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