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XFEM Simulation of Fatigue Crack Propagation in Peizoelectric Materials

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Abstract: (In this paper crack propagation under combined fatigue loading (mechanical and electrical) has been analyzed in a piezoelectric material (PZT4). XFEM (extended finite element method) together with level set method has been used for enrichment and solving the problem. Problems of edge crack and centre cracks in a rectangular plate have been considered. Interaction integral approach has been applied to determine stress intensity factor and the crack propagation criteria is checked to predict propagation and failure

Keywords – Crack propagation, piezoelectric materials

1 Introduction:

Piezoelectric materials have characteristics of mechanical to electrical inversion and vice versa and due to this property of piezoelectric material they are widely used in actuators, sensors etc. This property of piezoelectric material has got wide attention on the research work and many research works have been concentrated on piezoelectric materials and their behavior under applied mechanical and electrical loadings.

However, piezoelectric materials are brittle in nature, due to which they are prone to sudden failure. This brittle nature has got researchers attention to critically understand the reliability of piezoelectric devices and the failure behavior of the piezoelectric materials. In the past several investigations have been carried on brittle fracture of piezoelectric materials. Smart ceramics has been investigated for brittle fracture in [15]. Presence of void, holes, and cracks leading to stress concentration and can further reduce the life of piezoelectric material devices. Recently significant research has been focused on analysis of cracks in the piezoelectric materials. Near tip stress field and stress intensity factors have been analyzed in [14]. Effects of impact loading on cracks in piezoelectric materials have been analyzed [6]. Problems on Interface and sub interface cracks have also been addressed for piezoelectric materials [2, 15]. Investigation has also been carried out to develop fracture criteria for piezoelectric materials [5]. These Fracture criteria have been developed based on equivalent stress intensity factor and energy release rate.

Most of the above work addressed above has used one or other numerical methods for carrying out the analysis. Different numerical methods i.e. BEM (Boundary Element Method), FDM (Finite difference method), FEM (Finite Element Method) have been applied in fracture mechanics simulation and analysis of cracks in piezoelectric materials. Recently, extended finite element method (XFEM) [10] has been developed using partition of unity enrichment technique in finite element method. XFEM has proven to be an efficient tool in solving crack problems without remeshing, as in conventional finite element method. In XFEM enrichment functions are used to model discontinuities. X-FEM has been widely used in analysis of crack in fracture mechanics. It has been for fatigue life estimation of Functionally Graded Materials (FGM) [4]. XFEM has also been applied to simulation of crack in piezoelectric materials [1,3]. Sub interface crack for piezoelectric biomaterials has been analyzed using XFEM [12].

In the present work, XFEM has been applied for analysis of fatigue crack propagation in 2D for piezoelectric materials under combined (mechanical and electrical) loadings. Edge crack and centre crack problems have been considered for the analysis. Enrichment functions formulated [1] have been used for piezoelectric materials together with the crack propagation criteria for predicting failure.

2. XFEM Formulation for Piezoelectric Materials

For piezoelectric materials the field variables are displacement vector u_i and electric potential χ_j Mechanical strain tensor ε_{ij} and electric field vector E_i are deduced as [7]

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}); \quad E_i = -\chi_j \tag{1}$$

In the absence of body forces, σ_{ij} Cauchy stress tensor, D_i is the electric displacement vector are given by

$\sigma_{ij,j}=0,\ D_{i,j}=0$ on domain (2)

and are subjected to boundary condition on surface

$$\sigma_{ij}n_j = T_j^0 \quad D_j n_j = \varpi^0 \quad u_j = u_j^0 \quad \chi = \chi^0 \qquad (3)$$

Superscripts 0 stands for known value.

Crack surfaces considered to be traction free.
For homogeneous piezoelectric material constitutive relations are given by

$$\sigma_{ij} = C_{ijks}\varepsilon_{ks} - \xi_{sij}E_s \ \text{ And } \ D_i = \xi_{iks}\varepsilon_{ks} + z_{is}E_s \qquad (4)$$

Where C_{ijkl} , ξ_{iks} , z_{is} are the elasticity constants, piezoelectric constants and dielectric permittivity respectively

Above relations and boundary conditions are used to determine the displacement vector u_i and electric potential χ_j

Enriched approximation

XFEM approach together with level set method is applied for enrichment and mechanical displacement and electric potential are given by

$$u^h(x) = \sum_{i\in t_n} S_i(x)u_i + \sum_{j\in s_n} S_j(x)(h(f^h(x)) - h(f_j))\lambda_j^c + \sum_{l\in tip_n} S_l(x)\sum_{k=1}^{6}(A_f(r,\theta,\omega_k^{re},\omega_k^{im}) - A_f(x,\omega_k^{re},\omega_k^{im}))\lambda_l^t \qquad (5)$$

$$\chi^h(x) = \sum_{i\in t_n} S_i(x)\chi_i + \sum_{j\in s_n} S_j(x)(h(f^h(x)) - h(f_j))\gamma_j^c + \sum_{l\in tip_n} S_l(x)\sum_{k=1}^{6}(A_f^k(r,\theta,\omega_k^{re},\omega_k^{im}) - A_f^k(x,\omega_k^{re},\omega_k^{im}))\gamma_l^t \qquad (6)$$

Where S_i is shape function associated with node i.
$\lambda^c, \lambda^t, \gamma^c, \gamma^t$ are the enriched degrees of freedom associated with the crack elements. $h(f)$ is heaviside function and A_f^k represents asymptotic crack tip enrichment functions

Six fold enrichment function proposed in [1] is used. Interaction integral considering two equilibrium states of for cracked body is used. The first state is the actual state while the second one is the auxiliary state near the crack tip. Superposition of the two gives

$$J = J^{(1)} + J^{(2)} + I^{(1,2)} \qquad (7)$$

Where $J^{(1)}$ and $J^{(2)}$ are the electromechanical J integrals for state actual (1) and auxiliary (2) state and

$$I^{(1,2)} = \int_A \begin{pmatrix} \sigma_{ij}^{(1)}\dfrac{\partial u_i^{(2)}}{\partial x_1} + D_j^{(1)}\dfrac{\partial \chi^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)}\dfrac{\partial u_i^{(1)}}{\partial x_1} + \\ D_j^{(2)}\dfrac{\partial \chi^{(1)}}{\partial x_1} - \tilde{Q}^{(1,2)}\delta_{1j} \end{pmatrix} \frac{\partial w}{\partial x_j}\, dA \qquad (8)$$

in which

$$\tilde{Q}^{(1,2)} = \frac{1}{2}(\sigma_{ij}^{(1)}\varepsilon_{ij}^{(2)} - D_j^{(1)}E_j^{(2)} + \sigma_{ij}^{(2)}\varepsilon_{ij}^{(1)} - D_j^{(2)}E_j^{(1)}) \qquad (9)$$

Above equations are used to determine stress intensity factors.

3. Fatigue Crack Growth Criteria

Intensity factor obtained are used to find out equivalent or generalized stress intensity factor [5]

$$K_{eq} = A'K_I^{(1)} + B''K_{IV}^{(1)} \qquad (10)$$

Where A' and B'' depends on material and can be find out as per [5].
Crack propagation will takes place till $K_{eq} < K_{IC}$ fracture toughness.

Crack growth direction θ_c is determined on the basis on normal stress theory and is given by

$$\theta_c = 2\tan^{-1}\left(\frac{K^{(1)}_I - \sqrt{{K^{(1)}_I}^2 - 8{K^{(1)}_{II}}^2}}{4K^{(1)}_{II}}\right)$$

4. Problem Description, Results and Discussion:

In the present work problem of edge crack and centre crack has been taken as shown in **Fig. 1(a)** and **Fig. 1(b)** and are investigated for crack propagation of piezoelectric materials considering combined loading (mechanical and electrical). Fracture criteria established in [5] has been considered for propagation of cracks till fracture.

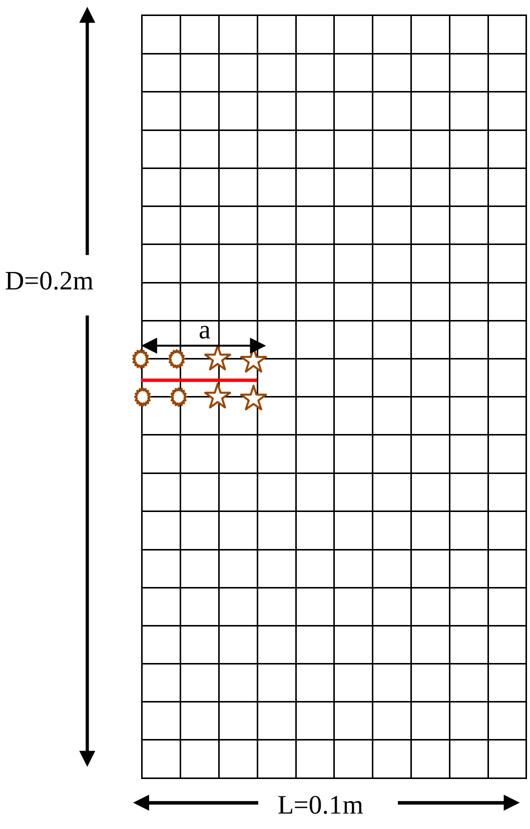


Fig. 1(a) Edge crack body

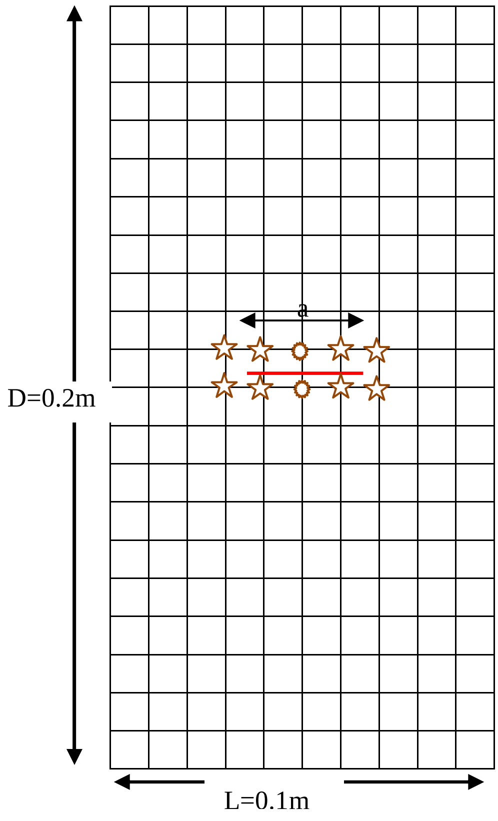


Fig. 1(b) Centre crack body

Simulation of crack model is done by applying XFEM together with level set method. Six basis enrichment functions are used for enriched nodes as formulated below. Stress intensity factor is evaluated at each iteration and is compared with fracture toughness to predict fracture.

Piezoelectric material properties shown in **Table 1** for PZT4 [5] has been considered for purpose of analysis.

Table 1: Pzt-4 material Properties [5]

Material Properties	**PZT4**
c_{11} (N/m^2)	13.9e+10
c_{12} (N/m^2)	7.78e+10
c_{13} (N/m^2)	7.43e+10
c_{33} (N/m^2)	11.3e+10
c_{44} (N/m^2)	2.56e+10
c_{66} (N/m^2)	3.06e+10
e_{31} (C/m^2)	-6.98
e_{33} (C/m^2)	13.8
e_{15} (C/m^2)	13.4
z_{11} C/Vm	6e-09
z_{33} C/Vm	5.47e-09
Fracture toughness K_{IC} MPa√m	2

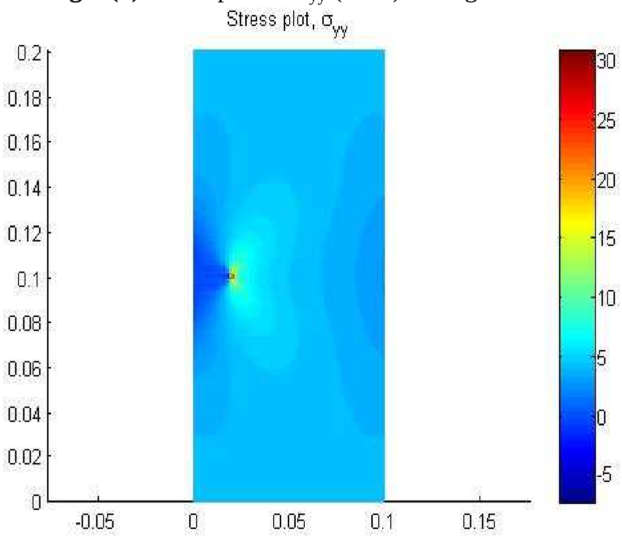
4.1 Edge crack propagation

A rectangular body of Length $L = 0.1m$ and height $D = 0.2m$ with initial crack length of $a = 0.02m$ has been considered for the analysis; a uniform mesh of 100 nodes in **X**-direction and 180-nodes in **Y**-direction is applied. Below loads have been applied on edge crack body. At each step crack extension of 1/5th of initial crack length has been considered for evaluating stress intensity factor

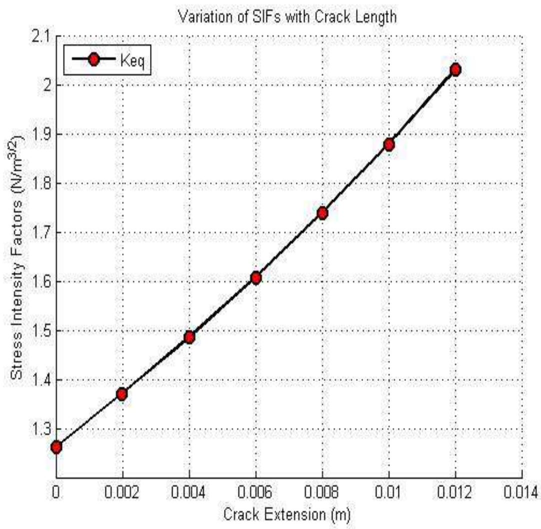
1. Mechanical load of 4*MPa* during mechanical load analysis
2. Electrical 1e-08 *volt/m* during combined loading together with the mechanical load

Fracture toughness considered for analysis $K_{Ic} = 2MPa\sqrt{m}$ [9]

Fig. 2(a) Stress plot of σ_{yy} (MPa) during Combined



loading**Fig. 2(b)** Plot of Stress Intensity Factor for edge crack v/s crack extension



During combined loading of edge crack rectangular body, **Fig. 2(a)** shows the variation of stress σ_{yy} acting along the y-direction during the 1st step. The maximum stress obtained is near about 30 *MPa* near the crack tip. **Fig. 2(b)** shows variation of stress intensity factor K_{eq} for Edge crack with crack extension in combined loading. In this case also the crack extends by near about *0.012m* before the ultimate fracture of the rectangular body.

4.2 Centre Crack Propagation

For centre crack body same dimension of the body has been considered as shown in **Fig. 1(b)** . Length $L = 0.1m$ and height $D = 0.2m$ with initial crack length of $a = 0.02m$ with a uniform mesh of 100 nodes in **X**-direction and 180 nodes in **Y**-direction. Same loads have been applied on centre crack body. At each step crack extension of 1/5th of initial crack length has been considered for evaluating stress intensity factor

1. Mechanical load of 4*MPa* during mechanical load analysis
2. Electrical 1 e-08 *volt/m* during combined loading together with the mechanical load

Fracture toughness considered for analysis $K_{Ic} = 2MPa\sqrt{m}$ [9]

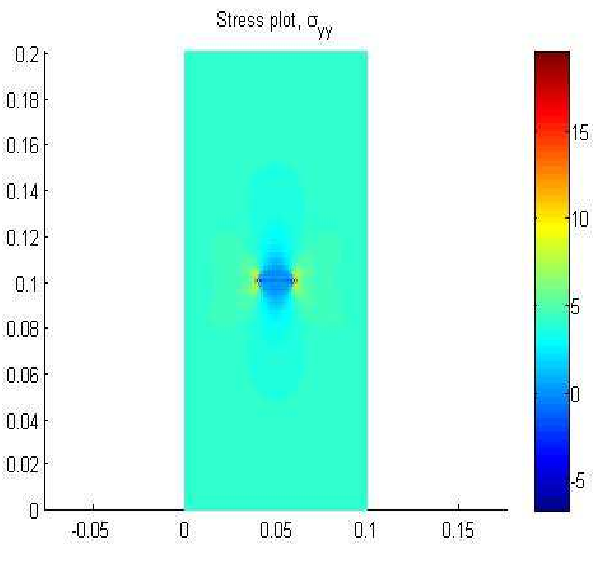


Fig. 3(a) Stress plot of σ_{yy} (MPa) during Combined loading

Above **Fig. 3(a)** shows the plot of σ_{yy} acting along the y-direction on the centre crack body at the 1st step of combined loading. Maximum stress observed for centre crack above is 15 *Mpa* which is lower as compared to 30*Mpa* observed for edge crack body

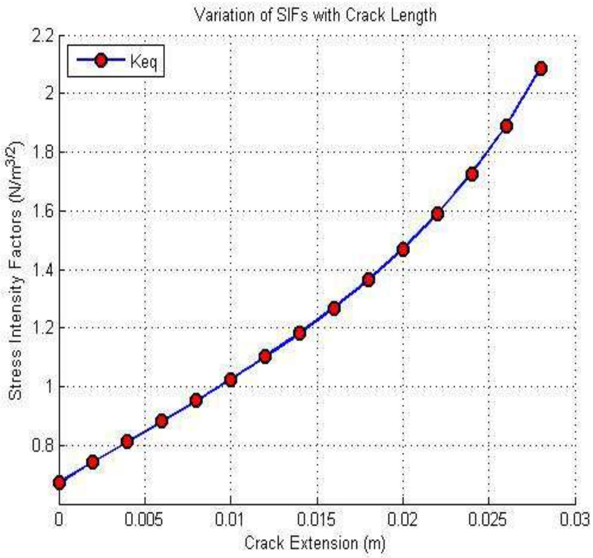


Fig. 3(b): Plot of Stress Intensity Factor for centre crack v/s crack extension

Fig. 3(b) shows variation of stress intensity factor for centre with crack extension in combined loading. Crack extension observed to be 0.027m.

5 Conclusions

In the present work fatigue crack growth analysis of the piezoelectric material PZT 4 is performed using XFEM for 2D edge crack and centre crack bodies. Cases of Combined (mechanical and electrical) loading have been considered in the analysis. On the basis of results in the present analysis it is observed that under combined loading, for the same crack length and same rectangular body centre cracks has lower stresses at the crack tip and also low stress intensity factor as compared to edge crack body. It can be also found that edge crack body will fail early as compared to centre crack body of the same dimension.

References

[1] Bechet E, Scherzer M, Kuna M., "Application of the X-FEM to the fracture of piezoelectric materials", Int J Numer Meth Engg, 2009;77:1535–65.
[2] Beom HG, Atluri SN., "Near-tip fields and intensity factors for interfacial cracks in dissimilar anisotropic piezoelectric media", Int J Fract, 1996;75:163–83.

[3] Bhargava RR, Sharma K., "A study of finite size effects on cracked 2-D piezoelectric media using extended finite element method", Comput Mater Sci, 2011;50:1834–45.
[4] Bhattacharya S, Singh I. V, Mishra B. K., "Fatigue-life estimation of functionally graded materials using XFEM", Engineering with Computers, (2013) 29:427–448
[5] Daining Fang , Zhen-Ke Zhang , Ai Kah Soh , Kwok Lun Lee , "Fracture criteria of piezoelectric ceramics with defects", Mechanics of Materials, 36 (2004) ;917–928
[6] Hu Shuling, Shen Shengping, Nishioka Toshihisa, "Numerical analysis for a crack in piezoelectric material under impact", International Journal of Solids and Structures, 44 (2007) 8457–8492
[7] Kuna M., "Finite element analyses of crack problems in piezoelectric structures", Computational Material Science, 1998;13:67–80.
[8] Kuna M,. "Finite element analyses of cracks in piezoelectric structures: a survey", Arch Appl Mech, 2006;76:725–45.
[9]Kuna M., "Fracture mechanics of piezoelectric materials–where are we right now?", Engg Fract Mech, 2010;77:309–26.
[10] Moes N, Dolbow J, Belytschko T., "A finite element method for crack growth without remeshing", Int J Numer Meth Engng 1999;46:131–50.
[11] Rao BN, Kuna M., "Interaction integrals for fracture analysis of functionally graded magnetoelectroelastic materials", Int J Fract, 2008;153:15–37.
[12] Sharma K., "Analysis of a subinterface crack in piezoelectric biomaterials with the extended finite element method", Engg Fract Mechanics, 104 (2013) 114–139.