Weighted Goal Programming Multiple Non-Linear Regression Model with Two-way Interaction Effect

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Abstract- Weighted Goal programming has been proven a valuable mathematical programming form in a number of venues. Weighted Goal Programming is the most widely used approach in the field of multiple criteria decision making that enables the decision maker to incorporate numerous variations of constraints and goals. Weighted Goal programming model serves a valuable purpose of cross checking answers from other methodologies. Different computer programmes are used to solve these GP models. Likewise, multiple regression models can also be used to more accurately combine multiple criteria measures that can be used in GP model parameters. Those parameters can include the relative weighting and the goal constraint parameters. This paper gives a brief review of the Weighted Goal Programming Multiple Non-Linear Regression Model with Interaction Effect.

Keywords: Weighted Goal Programming, Multiple Regression, Least Square Method, Interaction effect

1. INTRODUCTION
Regression analysis is used to understand the statistical dependence of one variable on other variables. Linear regression is the oldest and most widely used predictive model in decision making in managerial sciences, environmental science, and all the areas wherever it is required to describe possible relationships between two or more variables.

Regression analysis is used to understand the statistical dependence of one variable on other variables. Linear regression is the oldest and most widely used predictive model in decision making in managerial sciences, environmental science, and all the areas wherever it is required to describe possible relationships between two or more variables. This technique can show what proportion of variance between variables is due to the dependent variable, and what proportion is due to the independent variables. The linear regression can be classified into two types, simple linear regression and multiple linear regressions (MLR). The simple linear regression describes the relationship between two variables and MLR analysis describes the relationship between several independent variables and a single dependent variable. A number of methods for the estimation of the regression parameters are available in the literature. These include methods of minimizing the sum of absolute residuals, minimizing the maximum of absolute residuals and minimizing the sum of squares of residuals [13], where the last method of minimizing the sum of squares of residuals popularly known as least square methods is commonly used. Alp et al. [17] explained that linear goal pro-programming can be proposed as an alternative of the Least Square Method. For this he took an example of vertical network adjustment. Hassonpour et. al. [14] pro-posed a linear programming model based on goal programming to calculate regression coefficient. Saha [20] used the binary logistic regression model to analyze the school examination result (scores) of 1002 students.

An interaction occurs when the magnitude of the effect of one independent variable varies as a function of a second independent variable. This is also known as a moderation effect, although some have more strict criteria for moderation effects than for interactions. Now days interaction effects through regression models is a widely interested area of investigation as there has been a great deal of confusion about the analysis of moderated relationships involving continuous variables. Alken and West [1] have analyzed such interaction effects, further this method was applied into several models by the researchers, for exam-ple, Curran et. al. [6] applied into hierarchical linear growth models.

Multiple Objective optimization techniques provides more realistic solutions for most of the problems as it deals with multiple objective whereas single objective optimization techniques provides solutions to the problems that deals with single objective. Goal programming (GP) is a type of multiple objective optimization technique that converts a multi objective optimization model into a single objective optimization model. GP model has been proven a valuable tool in support of decision making. The first publication using GP was the form of a constrained re-gression model was used by Charnes et al. [4]. There have been many books devoted to this topic over past years (Ijiri [10]; Lee [11]; Sprouk [12]; Ignizio [9] and others). This tool often represents a substantial improvement in the modeling and analysis of multi-objective problems (Charnes and Cooper [4]; Eiselt et al [7]; Ignizio
2. MULTIPLE REGRESSION

Multiple regression is a technique that allows additional factors enter in the regression analysis separately, so that the effect of each can be estimated. In other words a linear regression model that contains more than one predictor variable is called a multiple regression model.

Let $X_{i0} = 1$ for $i = 1, 2, ..., n$. Let $X_{i1}, X_{i2}, ..., X_{im}$ be $m$ independent variables. Then a linear relationship can be modeled as:

$$Y_i = \sum_{j=0}^{m} b_j X_{ij} + e_i$$

Where $b_0, b_1, ..., b_m$ are the parameters to be estimated and $e_i$ is the error components which are assumed to be normally and independently distributed with zero mean and constant variance.

The linear absolute residuals method requires us to estimate the values of unknown parameters $b_0, b_1, ..., b_m$ so as to minimize

$$\sum_{i=1}^{n} |Y_i - \hat{Y}_i|$$

Where

$$\hat{Y}_i = \sum_{j=0}^{m} b_j X_{ij}$$

and $j = 0, 1, ..., m$

Where estimated values of the unknown parameters are represent by $\hat{b}_j$.

The least squares principle requires us to choose $b_0, b_1, ..., b_m$ which make

$$\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

least.

2.1 Interaction effect

Gupta et. al. [19] solved a multi objective investment management planning problem using fuzzy min sum weighted fuzzy goal programming technique. Application of a multi-objective programming model like goal programming model is an important tool for studying various aspects of management systems (Sen and Nandi [20]). As an extension to the findings of Sharma et. al. [21], the paper is focused on study of the Weighted Goal Programming Multiple Non-Linear Regression Model with two way Interaction Effect.

Interaction effect represents the combined effect of variables on the criterion or dependent measure. In other words an "interaction effect" has traditionally implied a separate effect of an independent variable on the dependent variable. The "product term" actually represents a portion of the effect of the independent variables on the dependent variable.

An interactive model can be represented as:

$$Y = a_0 + a_1(X_{i1}) + a_2(X_{i2}) + a_3(X_{i1}X_{i2})$$

In equation $a_3$ represents the interaction effect between $X_{i1}$ and $X_{i2}$ independent variables, which is referred to here as the "product term". In a traditional linear regression model (without a product term), the slope of $Y$ on $X_{i1}$ has a constant value across all values of $X_{i2}$.

3. REGRESSION FORMULATION

The regression equation used to analyze and interpret a two- way interaction effect, represent

$$Y_i = a_0 + \sum_{j=1}^{m} a_j X_{ij} + \sum_{k=1}^{n} a_k X_{ik} + \sum_{l=1}^{n} a_{kl} X_{il} + e_i = Y_{ir}$$

where

$$i = 1, 2, ..., m$$

$$j = 0, 1, ..., m$$

$$k = n+1, ..., 2n$$

$$l = 2n+1, ..., 2n + \binom{n}{2}$$

Where $a_0, a_1, a_2, a_3, a_4, a_5, ..., a_j, ..., a_k, ..., a_{kl}$ are the parameters to be estimated, these are contribution of decision variables $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, ..., X_{il}, ..., X_{ik}, ..., X_{il}$, respectively, except $a_0$. Here $Y_{10}$ taken as dependent variable and $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, ..., X_{il}$ are first and $n$th independent variable in linear form. Square of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, ..., X_{ik}$ (nonlinear form) taken in linear form as $X_{ik}$, also product term of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, ..., X_{il}$ (representing interaction effect) taken in linear form as $X_{il}$ respectively.

The linear absolute residual method requires us to estimate the values of unknown parameters so as to minimize

$$\sum_{i=1}^{n} |Y_i - \hat{Y}_{ir}|$$
4. WEIGHTED GOAL PROGRAMMING FORMULATION

Let $Y_{ig}$ be the $i$th goal, $d^+_i$ be positive deviation from the $i$th goal and $d^-_i$ be the negative deviation from the $i$th goal. $w^+_i$ and $w^-_i$ represent weights, those reflect the decision maker’s preferences regarding the relative importance of each goal.

Then the problem of Minimize $\sum_{i=1}^{m}|Y_i - Y_{ig}|$ may be reformulated as:

\[
\text{Minimize} \quad \sum_{i=1}^{m} (d^+_i + d^-_i)
\]

Subject to:

\[
a_0 + \sum_{j=1}^{n} a_j X_{ij} + \sum_{k=n+1}^{m} a_k X_{ik} + \sum_{l=2n+1}^{m+1} a_l X_{il} + d^+_i - d^-_i = Y_{ig}
\]

Non-negativity constraint,

\[
x_{ij}, x_{ik}, x_{il}, x_{ij}, \ldots \ldots \ldots x_{ik}, \ldots \ldots x_{il} \geq 0,
\]

\[
d^+_i \geq 0, \quad d^-_i \geq 0
\]

Complementary constraints

\[
d^+_i \times d^-_i = 0
\]

Where \(a_0, a_1, a_2, a_3, a_4, \ldots \ldots a_j, \ldots \ldots, a_k \)

..... are the parameters to be estimated, these are contribution of decision variables $X_{ij}, X_{ik}, X_{il}, x_{ij}, \ldots \ldots x_{ik}, \ldots \ldots x_{il}$ respectively, except $a_0$. Here $Y_{ig}$ taken as dependent variable and $X_{ij}, X_{ik}, X_{il}, X_{ij}, \ldots \ldots X_{ik}, \ldots \ldots X_{il}$ respectively, first and second and nth independent variable in linear form. Square of $X_{ij}, X_{ik}, X_{il}, X_{ij}, \ldots \ldots X_{ij}$ (nonlinear form) taken in linear form as $X_{ik}$ also product term of $X_{ij}, X_{ik}, X_{il}, X_{ij}, X_{ik}, \ldots \ldots X_{ij}$ (representing interaction effect) taken in linear form as $X_{il}$ respectively, to formulate the multiple nonlinear regression problem with two way interaction effect into linear goal programming model.

5. CONCLUSION

Over the last 30 years, GP Multiple Regression problems have been deployed extensively. This paper has briefly reviewed many of the highlights. There is a huge capacity for future developments and applications of GP.

The weighted goal programming technique provides the users with a better degree of estimates of the Multiple Non-Linear Regression parameters with two - way Interaction Effect. Solution of various problems related to estimation in science and technology obtained through presented weighted goal programming technique using various software packages like Microsoft Office Excel, LINGO, LINDO and TORA.

REFERENCES


