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Solving Fuzzy Volterra Integro-Differential Equations by Using Fuzzy Kamal Transform

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Abstract—In this paper, the fuzzy Kamal transform is used to solve fuzzy Volterra integral-differential equations, which is based on Kamal transform. Kamal transform takes very little computation and time. Numerical examples are given to prove the effectiveness of Kamal transform in solving fuzzy Volterra integro-differential equations.

Index Terms—convolution theorem, fuzzy numbers, fuzzy Volterra integro-differential equations, fuzzy Kamal transform.

I. INTRODUCTION

The initial conditions given by integral-differential equation models are usually unambiguous, and in fact, the variables involved in the related problems are uncertain. Therefore, fuzzy concepts are applied for integral and differential equations. Zadeh first introduced the concepts of fuzzy sets and set operations [1]. Later, Dubois and Prade proposed the fuzzy concept of integration [2–4] by using the extension principle. The definition of Hukuhara differentiability was used to solve fuzzy differential equations [5–6]. Goetschel and Voxman developed the concept of Rieman integral [7]. Friedman solved the numerical solutions of fuzzy integral equations and fuzzy differential equations [8]. Later, fuzzy integral equations and fuzzy differential equations were extended to the field of fuzzy integral-differential equations [9]. One type of fuzzy integral-differential equations is fuzzy Volterra integral-differential equation, which is involved in science, biology, physics, chemistry and other fields. Hajighasemi and Allahviranloo studied the existence and uniqueness of solutions of fuzzy Volterra integro-differential equations [10].

There are some different numerical methods to solve fuzzy Volterra integral-differential equation. Mikaeilvand et al. [11] used differential transform method (DTM) to solve fuzzy integro-differential equation in the form.

$$u'(\mathrm{x}) = \mathrm{f}(\mathrm{x}) + \int_0^{\mathrm{x}} \mathrm{k}(\mathrm{x},\mathrm{t})\mathrm{u}(\mathrm{x})\, dt,$$

where f(x) is a given function, k(t,x) is a known real-valued integral kernel and $\mathrm{x} \in [a, \mathrm{b}], \mathrm{b} < \infty$.

Manmohan and Talukdar solved the following fuzzy Volterra integral-differential equation by using the fuzzy Laplace transform [12].

$$u^{(n)}(t) = f(t) + \int_0^t k\,(s-t)u(s)\,d\,(\,s\,), \qquad (1.1)$$

$$u^{(k)}(0) = a_k = \left(\underline{a_k},\ \overline{a_k}\right),\ 0 \le k \le n-1. \qquad (1.2)$$

Rajkumar and Jesuraj solved fuzzy linear Volterra integro-differential equation by using the fuzzy Sumudu transform [13].

$$\begin{cases} y'(t) = x(t) + \lambda \int_a^t k(t,\tau)y(\tau)\,d\tau,\ \lambda > 0,\ a \le t \le b, \\ y(t_0) = \left(\underline{y}_\alpha,\ \overline{y}_\alpha(0)\right),\ 0 < \alpha \le 1, \end{cases}$$

where $\mathrm{k}(\mathrm{t},\tau)$ is an arbitrary real valued kernel function, the initial condition is a generalized triangular fuzzy number.

Majid and Rabiei [9] proposed a fuzzy general linear method of order three to solve fuzzy Volterra integro-differential equations of second kind in the form

$$y'(t) = f(t,y) + \int_0^X K(t,s)y(s)\,ds,\ y(t_0) = y_0,$$

where y_0 is a fuzzy number.

Kamal transform was introduced by Abdelilah Kamal. It is derived from classical Fourier integral and it could be applied to the solution of differential equations in the time domain. In 2016, Abdelilah and Hassan solved the ordinary differential equation by using the Kamal transform [14]. In 2018, Aggarwa and Chauhan solved the linear Volterra integral equation by using the Kamal transform [15]. Aggarwal and Gupta successfully used the Kamal transform to solve the first kind and second kind of linear Volterra integral equations [16–17], and this method takes very little computation and time. In this paper, the Kamal transform will be applied to solve Eq (1.1) and Eq (1.2).

The work is organized as following. Section 2 introduces some basic definitions and theorems about fuzzy numbers and Kamal transform. Section 3 gives the fuzzy Kamal transform method for solving fuzzy Volterra integral-differential equations. Section 4 illustrates examples to demonstrate the effectiveness of this method. Finally, Section 5 is a brief conclusion.

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II. PRELIMINARIES

This section is devoted to the definitions and some theorems regarding fuzzy numbers and Kamal transform.

Definition2.1. [18] Given a nonempty set X, a fuzzy subset A is characterized by a membership function $f_A(x): X \to [0,1]$ which represents "the grade of membership" of X in A.

Definition2.2. [19–21] A fuzzy number $\tilde{u}$ can be represented as $\tilde{u} = [\underline{u}(r), \overline{u}(r)], 0 \le r \le 1$, u satisfies the following conditions.

(i) $\underline{u}(r)$ is a left-continuous, bounded, monotonic increasing function.

(ii) $\overline{u}(r)$ is a left-continuous, bounded, monotonic decreasing function.

(iii) $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

If $\underline{u}(r) = \overline{u}(r) = r$, then r is a crispy number.and crisp number j, For two fuzzy numbers $\tilde{u} = [\underline{u}(r), \overline{u}(r)], \tilde{v} = [\underline{v}(r), \overline{v}(r)]$ and crisp number j , addition and scalar multiplication are defined as

(1) $(\underline{u+v})(a) = (\underline{u}(a) + \underline{v}(a))$.

(2) $(\overline{u+v})(a) = (\overline{u}(a) + \overline{v}(a))$.

(3) $(j\underline{u})(a) = j\underline{u}(a), (j\overline{u})(a) = j\overline{u}(a), j \ge 0$.

(4) $(j\underline{u})(a) = j\overline{u}(a), (j\overline{u})(a) = j\underline{u}(a), j < 0$.

Let L(R) be the Banach space that contains all the integrable functions in R.

Definition2.3. [22] If $f \to L(R)$ is a fuzzy value function, for $x_0 \in R$, there is a $f'(x_0) \in L(R)$ that can be given by

(1) $\lim_{h \to 0^+} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h \to 0^+} \frac{f(x_0)-f(x_0-h)}{h} = f'(x_0)$.

(2) $\lim_{h \to 0^-} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h \to 0^-} \frac{f(x_0)-f(x_0-h)}{h} = f'(x_0)$.

Theorem2.1. [23] If $f \to L(R)$ is a fuzzy value function, f(t) can be expressed as $f(t) = [\underline{f}(t,a), \overline{f}(t,a)]$, there are two types of $f'(t)$ here as follows

(a) If f is differentiable defined by (1) in Definition 2.3, then

$$f'(t) = [\underline{f'}(t,a), \overline{f'}(t,a)],$$

(b) If f is differentiable defined by (2) in Definition 2.3, then

$$f'(t) = [\overline{f'}(t,a), \underline{f'}(t,a)].$$

Next we introduce the definitions and some theorems about Kamal transform.

Definition2.4. [14] Let K be the Kamal transform operator, the Kamal transform of the function F(t) is defined as

$$K\{F(t)\} = \int_0^\infty F(t) e^{\frac{-t}{v}} dt := G(v), \ t \ge 0, \ k_1 \le v \le k_2.$$

Suppose that F(t) for $t \ge 0$ is piecewise continuous and of exponential order, then the Kamal transform of the function F(t) exists. These are the sufficient conditions for the existence of the Kamal transform.

Theorem2.2. [14,15,17,24–27] The Kamal transform results of some elementary functions are list in Table 1.

Table 1:Kamal transform

S. N.	F(t)	K{F(t)} = G(v)
1.	1	v
2.	t	v^2
3.	t^2	$2!\, v^3$
4.	$t^n, n \in N$	$n!\, v^{n+1}$
5.	$t^n, n > -1$	$\tau(n+1) v^{n+1}$
6.	e^{at}	$\frac{v}{1-av}$
7.	$\sin at$	$\frac{av^2}{1+a^2v^2}$
8.	$\cos at$	$\frac{v}{1+a^2v^2}$
9.	$\sinh at$	$\frac{av^2}{1-a^2v^2}$
10.	$\cosh at$	$\frac{v}{1-a^2v^2}$

Theorem2.3. [14] If $K\{F(t)\} = G(v)$, the Kamal transform of $F'(t)$ satisfies

(1) $K\{F'(t)\} = \frac{1}{v} G(v) - F(0)$.

(2) $K\{F''(t)\} = \frac{1}{v^2} G(v) - \frac{1}{v} F(0) - F'(0)$.

(3) $K\{F^{(n)}(t)\} = \frac{1}{v^n} G(v) - \frac{1}{v^{n-1}} F(0) - \frac{1}{v^{n-2}} F'(0) \cdots \cdots - F^{(n-1)}(0)$.

Definition2.5. [15,25,26] If $K\{F(t)\} = G(v)$, then $F'(t)$ is defined as the inverse Kamal transform of G(t)

$$F(t) = K^{-1}\{G(v)\}, \tag{2.1}$$

with K^{-1} is the inverse Kamal transform operator.

Theorem2.4. [15,25,26] The inverse Kamal transforms of some elementary functions are list in Table 2.

Table 2:Inverse of Kamal transform

S. N.	F(t)	$F(t) = K^{(-1)}\{G(v)\}$
1.	v	1
2.	v^2	t
3.	v^3	$\frac{t^2}{2!}$
4.	$v^{n+1}, n \in N$	$\frac{t^n}{n!}$
5.	$v^{n+1}, n > -1$	$\frac{t^n}{\tau(n+1)}$
6.	$\frac{v}{1-av}$	e^{at}
7.	$\frac{v^2}{1+a^2v^2}$	$\frac{\sin at}{a}$
8.	$\frac{v}{1+a^2v^2}$	$\cos at$
9.	$\frac{v^2}{1-a^2v^2}$	$\frac{\sinh at}{a}$
10.	$\frac{v}{1-a^2v^2}$	$\cosh at$

Definition2.6. [24] The convolution of the two functions F(t) and G(v) are defined as

$$F(t) * G(t) = F * G = \int_0^t F(x) G(t-x)\, dx = \int_0^t F(t-x) G(x)\, dx. \tag{2.2}$$

Theorem2.5. [15,24] If $K\{F(t)\} = G(v)$ and $K\{H(t)\} = I(v)$, then

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$$K\{F(t)*H(t)\} = K\{F(t)\}K\{H(t)\} = G(v)I(v). \quad (2.3)$$

III. PROCEDUCE OF FUZZY KAMAL TRANSFORM

In this section, we will develop the version of Kamal transform to solve fuzzy Eq (1.1) and Eq (1.2).

Taking Kamal transform on both sides of Eq (1.1), we have

$$K\{u^n(t)\} = K\{f(t)\} + K\left\{\int_0^t k(s-t)u(s)\,ds\right\}. \quad (3.1)$$

Using Theorem 2.3, there is

$$\frac{1}{v^n}K\{u(t)\} = \frac{1}{v^{n-1}}u(0) + \frac{1}{v^{n-2}}u'(0) + \cdots\cdots + u^{(n-1)}(0) + K\{f(t)\} + K\left\{\int_0^t k(s-t)\,u(s)\,ds\right\}. \quad (3.2)$$

(3.2)

Then Theorem 2.5 gives

$$K\{u(t)\} = vu(0) + v^2u'(0) + \cdots\cdots + v^n u^{(n-1)}(0) + v^n K\{f(t)\} + v^n K\{k(t)\}K\{u(t)\}. \quad (3.3)$$

By using the initial condition Eq (1.2), Eq (3.3) can be expressed in the form of parameters as

$$K\{\underline{u}(t)\} = v\underline{a_0} + v^2\underline{a_1} + \cdots\cdots + v^n\underline{a_{n-1}} + v^n K\left\{\underline{f(t;a)}\right\} + v^n\underline{K\{k(t;a)\}K\{u(t;a)\}}, \quad (3.4)$$

$$K\{\overline{u}(t)\} = v\overline{a_0} + v^2\overline{a_1} + \cdots\cdots + v^n\overline{a_{n-1}} + v^n K\left\{\overline{f(t;a)}\right\} + v^n\overline{K\{k(t;a)\}K\{u(t;a)\}}. \quad (3.5)$$

Definition 2.2 deduces that $K\{k(t)\}K\{u(t)\}$ is generally divided into the following four cases.

Case 1: If $k(t,a) > 0$ and $u(t,a) > 0$, then

$$\underline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\underline{k(t;a)}\right\}K\left\{\underline{u(t;a)}\right\},$$

$$\overline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\overline{k(t;a)}\right\}K\left\{\overline{u(t;a)}\right\}.$$

Case 2: If $k(t,a) > 0$ and $u(t,a) < 0$, then

$$\underline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\overline{k(t;a)}\right\}K\left\{\underline{u(t;a)}\right\},$$

$$\overline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\underline{k(t;a)}\right\}K\left\{\overline{u(t;a)}\right\}.$$

Case 3: If $k(t,a) < 0$ and $u(t,a) > 0$, then

$$\underline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\underline{k(t;a)}\right\}K\left\{\overline{u(t;a)}\right\},$$

$$\overline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\overline{k(t;a)}\right\}K\left\{\underline{u(t;a)}\right\}.$$

Case 4: If $k(t,a) < 0$ and $u(t,a) > 0$, then

$$\underline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\overline{k(t;a)}\right\}K\left\{\overline{u(t;a)}\right\},$$

$$\overline{K\{k(t;a)\}K\{u(t;a)\}} = K\left\{\underline{k(t;a)}\right\}K\left\{\underline{u(t;a)}\right\}.$$

We first give the expressions of the solutions of Eq (1.1) and Eq (1.2) for Case 1. Now Eq (3.4) and Eq (3.5) in Case 1 become

$$K\{\underline{u}(t)\} = v\underline{a_0} + v^2\underline{a_1} + \cdots\cdots + v^n\underline{a_{n-1}} + v^n K\left\{\underline{f(t;a)}\right\} + v^n K\left\{\underline{k(t;a)}\right\}K\left\{\underline{u(t;a)}\right\},$$

and

$$K\{\overline{u}(t)\} = v\overline{a_0} + v^2\overline{a_1} + \cdots\cdots + v^n\overline{a_{n-1}} + v^n K\left\{\overline{f(t;a)}\right\} + v^n K\left\{\overline{k(t;a)}\right\}K\left\{\overline{u(t;a)}\right\}.$$

Therefore $K\{\underline{u}(t;a)\}$ and $K\{\overline{u}(t;a)\}$ can be expressed as

$$K\{\underline{u}(t;a)\} = \frac{v\underline{a_0} + v^2\underline{a_1} + \cdots\cdots + v^n\underline{a_{n-1}} + v^n K\{\underline{f}(t;a)\}}{\left(1 - v^n K\left\{\underline{k(t;a)}\right\}\right)}, \quad (3.6)$$

$$K\{\overline{u}(t;a)\} = \frac{v\overline{a_0} + v^2\overline{a_1} + \cdots\cdots + v^n\overline{a_{n-1}} + v^n K\{\overline{f}(t;a)\}}{\left(1 - v^n K\left\{\overline{k(t;a)}\right\}\right)}. \quad (3.7)$$

Using the fuzzy inverse Kamal transform on both sides of the Eq (3.6) and Eq (3.7), we can easily get $\underline{u}(t;a)$ and $\overline{u}(t;a)$

$$\underline{u}(t;a) = \frac{\underline{a_0} + t\underline{a_1} + \cdots\cdots + \frac{t^{n-1}}{(n-1)!}\underline{a_{n-1}} + K^{-1}\left\{v^n K\{\underline{f}(t;a)\}\right\}}{K^{-1}\left\{1 - v^n K\left\{\underline{k(t;a)}\right\}\right\}}. \quad (3.8)$$

and

$$\overline{u}(t;a) = \frac{\overline{a_0} + t\overline{a_1} + \cdots\cdots + \frac{t^{n-1}}{(n-1)!}\overline{a_{n-1}} + K^{-1}\left\{v^n K\{\overline{f}(t;a)\}\right\}}{K^{-1}\left\{1 - v^n K\left\{\overline{k(t;a)}\right\}\right\}}. \quad (3.9)$$

Similarly, we can easily get the solutions for Case 2.

$$\underline{u}(t;a) = \frac{\underline{a_0} + t\underline{a_1} + \cdots\cdots + \frac{t^{n-1}}{(n-1)!}\underline{a_{n-1}} + K^{-1}\left\{v^n K\{\underline{f}(t;a)\}\right\}}{K^{-1}\left\{1 - v^n K\left\{\overline{k(t;a)}\right\}\right\}}, \quad (3.10)$$

and

$$\overline{u}(t;a) = \frac{\overline{a_0} + t\overline{a_1} + \cdots\cdots + \frac{t^{n-1}}{(n-1)!}\overline{a_{n-1}} + K^{-1}\left\{v^n K\{\overline{f}(t;a)\}\right\}}{K^{-1}\left\{1 - v^n K\left\{\underline{k(t;a)}\right\}\right\}}. \quad (3.11)$$

The solutions for Case 3 are

$$\underline{u}(t;a) = \frac{\sum_{i=1}^{n}\frac{t^{i-1}}{(i-1)!}\underline{a_{i-1}}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^n K\{\underline{f}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{\sum_{i=0}^{n-1}\overline{a_i}K^{-1}\left\{v^{n+i+1}K\{\underline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^{2n}K\{\overline{f}(t;a)\}K\{\underline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)}, \quad (3.12)$$

and

$$\overline{u}(t;a) = \frac{\sum_{i=1}^{n}\frac{t^{i-1}}{(i-1)!}\overline{a_{i-1}}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^n K\{\overline{f}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{\sum_{i=0}^{n-1}\underline{a_i}K^{-1}\left\{v^{n+i+1}K\{\overline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^{2n}K\{\underline{f}(t;a)\}K\{\overline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)}. \quad (3.13)$$

The solutions for Case 4 are

$$\underline{u}(t;a) = \frac{\sum_{i=1}^{n}\frac{t^{i-1}}{(i-1)!}\underline{a_{i-1}}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^n K\{\underline{f}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{\sum_{i=0}^{n-1}\overline{a_i}K^{-1}\left\{v^{n+i+1}K\{\overline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^{2n}K\{\overline{f}(t;a)\}K\{\overline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\overline{k(t;a)}\right\}K\left\{\underline{k(t;a)}\right\}\right)}, \quad (3.14)$$

and

$$\overline{u}(t;a) = \frac{\sum_{i=1}^{n}\frac{t^{i-1}}{(i-1)!}\overline{a_{i-1}}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^n K\{\overline{f}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{\sum_{i=0}^{n-1}\underline{a_i}K^{-1}\left\{v^{n+i+1}K\{\underline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)} + \frac{K^{-1}\left\{v^{2n}K\{\underline{f}(t;a)\}K\{\underline{k}(t;a)\}\right\}}{K^{-1}\left(1 - v^{2n}K\left\{\underline{k(t;a)}\right\}K\left\{\overline{k(t;a)}\right\}\right)}. \quad (3.15)$$

IV. NUMERICAL EXAMPLES

In this section, examples are given to demonstrate the effectiveness of the fuzzy Kamal transform for solving the

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fuzzy Volterra integral-differential equations.

Example 4.1 Consider the following fuzzy Volterra integro-differential equation

$$u'(t) = (a+1, 3-a)(1+t) + \int_0^t -u(x)\,dx, \quad (4.1)$$

$$u(0) = (0,0). \quad (4.2)$$

The kernel function for this problem is $k(t;a) = -1 < 0$.

Case 1: if $u(x)$ is positive. From Eq (3.12) and Eq (3.13), we can obtain the solutions

$$\underline{u}(t;a) = (a+1)\left[\frac{1}{2}(e^t + \sin(t) - \cos(t))\right] - (3-a)\left[\frac{1}{2}(e^t + \cos(t) - \sin(t)) - 1\right], \quad (4.3)$$

and

$$\overline{u}(t;a) = (3-a)\left[\frac{1}{2}(e^t + \sin(t) - \cos(t))\right] - (a+1)\left[\frac{1}{2}(e^t + \cos(t) - \sin(t)) - 1\right]. \quad (4.4)$$

Case 2: if $u(x)$ is negative. From Eq (3.14) and Eq (3.15), the solutions are the same as the results of (4.3) and (4.4).

Example 4.2 Consider the following fuzzy Volterra integro-differential equation in [12]

$$u''(t) = (a+2, 4-a)t + \int_0^t (t-x)u(x)\,dx, \quad (4.5)$$

$$u(0) = (a+1, 3-a), \ u'(t) = (a, 2-a). \quad (4.6)$$

The kernel function for this problem is $k(t;a) = t - x > 0$ for $x \in [0, t]$.

Case 1: if $u(x)$ is positive. From Eq (3.8) and Eq (3.9), we can obtain the solutions

$$\underline{u}(t;a) = (a+2)\frac{1}{2}(\sinh t - \sin t) + (a+1)\frac{1}{2}(\cos t + \cosh t) + \frac{a}{2}(\sin t + \sinh t), \quad (4.7)$$

$$\overline{u}(t;a) = (4-a)\frac{1}{2}(\sinh t - \sin t) + (3-a)\frac{1}{2}(\cos t + \cosh t) + (2-a)\frac{1}{2}(\sin t + \sinh t). \quad (4.8)$$

Case 2: if $u(x)$ is negative. From Eq (3.10) and Eq (3.11), we can obtain the same solutions as (4.7) and (4.8).

For this example, we obtain the same solutions obtained by using Fuzzy Laplace Transformation transform in [12].

V. CONCLUSION

In this paper, the fuzzy Kamal transform is used to solve the fuzzy Volterra integral-differential equations successfully, based on the categories of kernel functions and unknown functions, the fuzzy equation is divided into four different types. Examples are given to demonstrate the effectiveness of the method.

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