International Journal of Research in Advent Technology, Vol.10, No.3, September 2022
E-ISSN: 2321-9637
Available online at www.ijrat.org

Incorporation of Competition Among Retailers in a Supply Chain Model with Carbon Emission Reduction Technology

Subhankar Adhikari

Abstract—As days are progressing the effect of global warming increases rapidly through-out the world. It is a bigger threat for not only human being but also for the whole animal kingdom. If we look deeply into the reason behind global warming, there are some gases behind it. In this regard, the most harmful gas which is responsible for global warming is $\boldsymbol{Co_2}$. Emission of carbon from different industry helps in formation of $\boldsymbol{Co_2}$. So, utility of carbon emission reduction technology is indispensible for both developed and developing countries. Different Governments consider the act of reduction of carbon emission level by applying some technology as a social responsibility. Very often this technology is termed as Green technology. Also, in many situations upper bound of carbon emission level is specified to arisen more consciousness among the people. A mathematical model has been framed incorporating Green technology. Solution procedure under different structures is discussed. Numerical example has been provided illustrating this model.

Index Terms—Supply Chain, Price Competition, Multiple Retailers, Green Technology, Carbon Tax.

I. INTRODUCTION

Carbon emission is a major environmental issue for inhabitants of the earth. Government of different countries are framing rules and regulations to restrict carbon emission level. Even if they are imposing taxes for extra carbon emission from industries with respect to a pre-determined upper bound of emission level. Activities in supply chain are also affected by the event of carbon-emission.

N. Absi et al. [1] studied problem of lot-sizing with carbon emission constraints. Dye and Yang [2] developed model for perishable goods with demand varies as length of credit period under emission constraints. An et al. [3] compared Green credit financing with Trade credit financing in supply chain environment under the limits of carbon emission. The effect imposed by carbon tariff on global emission control was portrayed in the work of Fang et al. [4]. The usage of Green technology to reduce carbon emission in a vendor managed inventory system was studied by Bai et al. [5]. He et al. [6] examined direct and cross effects when consumer demand is sensitive with respect to selling price, reduced carbon emission level and delivery time. Lee and Choi [7] developed a supply chain model where both supplier and buyer were both exerting efforts for reducing emissions. In that work the demand is influenced by both members effort level. Sarkar et al. [8] showed that if quality of production improves then it would have a positive effect on reduction of carbon emission level. Mandal and Pal [9] incorporated two tier credit facility in a green supply chain environment. Pal et al. [10] determined optimal decisions in a dual channel green supply chain along with competitive attitude of chain members. They also considered different centralized and decentralized approaches.

Noteworthy works of Hovelaque and Bironneau[11], Sarkar et al.[12], Song et al.[13] and Li[14] in this direction should be mentioned.

II. BASIC ASSUMPTIONS WITH NOTATIONS

- Here the supply chain consists of a manufacturer (M) and two retailers $(R_1\ and\ R_2)$.
- Manufacturer produces quantity Q and sells Q_1 quantity to the retailer R_1 and Q_2 to R_2. So $Q = Q_1 + Q_2$.
- Wholesale price of the manufacturer is W to both of the retailers.
- Retailers are involved in competition among themselves based on selling prices.
- Manufacturer invest in Green technology to reduce carbon emission during production period. 1 is the level of carbon emission from each unit without applying green technology. After application of Green technology carbon emission level is reduced by $\in$. As a matter of fact, using Green technology carbon emission level reached at $1-\in$.
- Production cost of one unit of manufacturer is C_P
- Carbon tax per unit per level is C_E.
- Retailer R_1 sells product per unit at a price S_1 while R_2 is selling at a price S_2 to the customers.
- D_1 is the demand rate for the retailer R_1. It depends on both $S_1\ and\ S_2$. In fact, D_1 is a monotonic decreasing function of S_1 and monotonic increasing function of S_2. In simple words, own selling price has negative impact while competitors selling price has positive impact. Exact form of D_1 is given below.

Manuscript revised on 14 September, 2022 and published on 30 September, 2022
Subhankar Adhikari, *Department of Mathematics, Chandernagore College, Chandernagore, Hooghly, West Bengal, India, Pin-712136.*

doi: 10.32622/ijrat.103202204

$D_1 = D_1(S_1, S_2) = A_1 - A_2S_1 + A_3S_2 + A_4 \in$

- Similarly, D_2 is the demand rate for the retailer R_2. It is given by
  $D_2 = D_2(S_1, S_2) = F_1 - F_2S_2 + F_3S_1 + F_4 \in$
  Reduced carbon level has a positive impact on customer demand rate.
- L, L_1, L_2 are set up cost of manufacturer, retailer R_1 and retailer R_2 respectively.
- H, H_1, H_2 are holding cost per unit of manufacturer, retailer R_1 and retailer R_2 respectively.
- Decision variables for the manufacturer is production quantity (Q) and carbon emission reduction level $(\in)$.
- Decision variable for retailer R_1 is S_1.
- Decision variable for retailer R_2 is S_2.
- $\pi_M, \pi_{R_1}, \pi_{R_2}$ are profit functions cost of manufacturer, retailer R_1 and retailer R_2 respectively.
- Production rate of manufacturer is P which is constant and production time is T_P
- Cycle length of retailer R_1 is T_1 and that of retailer R_2 is T_2.
- I_M, I_{R_1}, I_{R_2} are inventory levels of manufacturer, retailer R_1 and retailer R_2 respectively at any time t.

III. FORMULATION OF MATHEMATICAL MODEL

Formulation from manufacturer's perspective

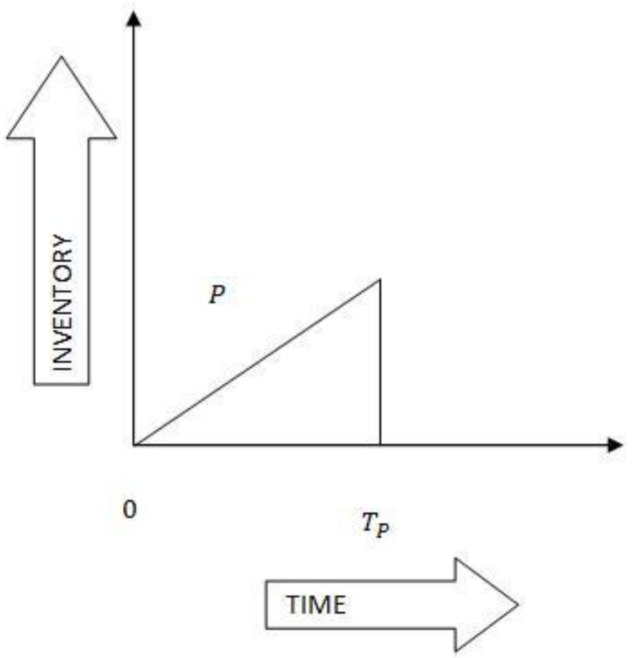

Figure1: Manufacturer time weighted inventory

The differential equation associated with manufacturer inventory level

$\frac{dI_M}{dt} = P$ subject to $I_M(0) = 0$ (1)

Solving this equation, we have manufacturer inventory level at any time t as

$I_M(t) = Pt$ (2)

Total holding cost of the manufacturer for the time interval $[0, T_P]$ is given below

$= H \int_0^{T_P} I_M(t)\, dt$

$= \frac{H}{2}\frac{Q^2}{P}$ [Using (2) and $T_P = \frac{Q}{P}$] (3)

Manufacturer average profit

$= \pi_M$

$= \frac{1}{T_P}[Earned\ sells\ revenue\ from\ retailers - Set\ up\ cost - Productioncost - Holding\ cost - Cost\ for\ investment\ in\ Green\ technology - Cost\ for\ carbon\ tax\ for\ extra\ carbon\ emission]$

$$= \frac{1}{T_P}\left[W\,Q - L - C_P\,Q - \frac{H}{2}\frac{Q^2}{P} - \frac{1}{2}\,k\,Q \in^2 - C_E\,(1-\in)\,Q\right] \quad (4)$$

Formulation from retailer R_1 end,

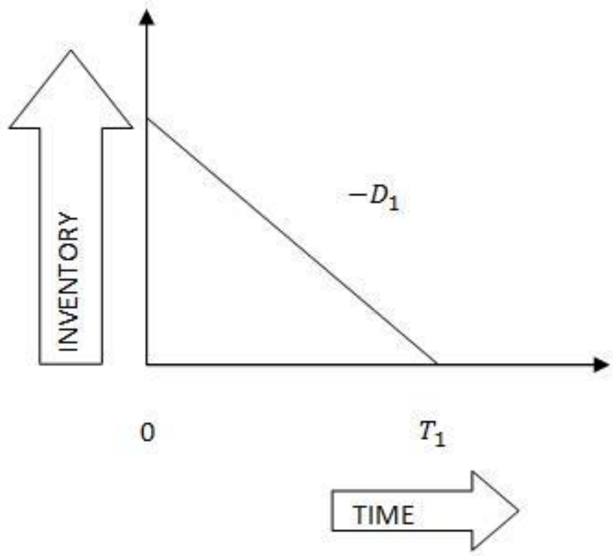

Figure 2: Retailer R_1 time weighted inventory

The differential equation governing retailer R_1 inventory level for the time interval $[0, T_1]$ is given by

$\frac{dI_{R_1}}{dt} = -D_1$ with $I_{R_1}(0) = Q_1$ (5)

Solution of the above differential equation gives

$I_{R_1}(t) = Q_1 - D_1\, t$ (6)

Using the terminal condition $I_{R_1}(T_1) = 0$, we get the cycle length of the retailer R_1 as

$T_1 = \frac{Q_1}{D_1}$ (7)

Holding cost for the retailer R_1 for the time interval $[0, T_1]$ is expressed as

$= H_1 \int_0^{T_1} I_{R_1}(t)\, dt$

$= \frac{H_1}{2}\,\frac{{Q_1}^2}{D_1}$

Retailer R_1 has the average profit

$= \pi_{R_1}$

doi: 10.32622/ijrat.103202204

$= \frac{1}{T_1}[Collected\ sales\ revenue - Set\ up\ cost - Holding\ cost]$

$$= \frac{1}{T_1}\left[Q_1(S_1 - W) - L_1 - \frac{H_1}{2}\frac{{Q_1}^2}{D_1}\right] \quad (8)$$

In a similar manner retailer R_2 has the profit function $= \pi_{R_2}$

$$= \frac{1}{T_2}\left[Q_2(S_2 - W) - L_2 - \frac{H_2}{2}\frac{{Q_2}^2}{D_2}\right] \quad (9)$$

IV. SOLUTION METHODOLOGY

This model can be framed in two approaches. One is centralized approach other is decentralized approach.

IV.A CENTRALISED APPROACH

In this approach the decision is made jointly by the members of the chain.

Let π_C denote the profit function of this approach. Then its expression is as follows

$$\pi_C = \pi_M + \pi_{R_1} + \pi_{R_2} \quad (10)$$

Solving four equations $\frac{\partial \pi_C}{\partial Q} = 0, \frac{\partial \pi_C}{\partial \in} = 0, \frac{\partial \pi_C}{\partial S_1} = 0, \frac{\partial \pi_C}{\partial S_2} = 0$ we get values of decision variables

$Q = Q^*, \in = \in^*, S_1 = {S_1}^*, S_2 = {S_2}^*$

The solution will be optimal if the eigen-values of the following Hessian matrix (HC_1) are negative.

$$HC_1 = \begin{bmatrix} \frac{\partial^2 \pi_C}{\partial Q^2} & \frac{\partial^2 \pi_C}{\partial Q\, \partial \in} & \frac{\partial^2 \pi_C}{\partial Q\, \partial S_1} & \frac{\partial^2 \pi_C}{\partial Q\, \partial S_2} \\ \frac{\partial^2 \pi_C}{\partial Q\, \partial \in} & \frac{\partial^2 \pi_C}{\partial \epsilon^2} & \frac{\partial^2 \pi_C}{\partial S_1\, \partial \in} & \frac{\partial^2 \pi_C}{\partial S_2\, \partial \in} \\ \frac{\partial^2 \pi_C}{\partial Q\, \partial S_1} & \frac{\partial^2 \pi_C}{\partial S_1\, \partial \in} & \frac{\partial^2 \pi_C}{\partial {S_1}^2} & \frac{\partial^2 \pi_C}{\partial S_1\, \partial S_2} \\ \frac{\partial^2 \pi_C}{\partial Q\, \partial S_2} & \frac{\partial^2 \pi_C}{\partial S_2\, \partial \in} & \frac{\partial^2 \pi_C}{\partial S_1\, \partial S_2} & \frac{\partial^2 \pi_C}{\partial {S_2}^2} \end{bmatrix} \quad (11)$$

IV.A DECENTRALISED APPROACH

There may be many decentralized approaches. Here in this work we discuss Manufacturer Stackelberg approach.

In this approach manufacturer is the leader and both retailers are followers. First followers determine their optimal strategy. Based on that manufacturer(leader) determine his optimal policy.

Retailer R_1 determine his optimal selling price$S_1 = {S_1}^*$ by solving the equation

$\frac{\partial \pi_{R_1}}{\partial S_1} = 0$ and he must check the second order condition

$\left.\frac{\partial^2 \pi_{R_1}}{\partial {S_1}^2}\right|_{S_1 = {S_1}^*} < 0$

In a similar fashion, retailer R_2 determine his optimal selling price $S_2 = {S_2}^*$.

Then manufacturer will optimize his profit function substituting $S_1 = {S_1}^*, S_2 = {S_2}^*$

V. NUMERICAL EXAMPLE

The values of parameter given in a tabular form.

Parameter	Values
A_1	400
A_2	9
A_3	4
A_4	8
F_1	300
F_2	12
F_3	7
F_4	5
W	25
L	2000
L_1	800
L_2	780
H	1
H_1	0.75
H_2	0.80
C_P	7
P	600
k	3
C_E	0.8
Q_1	0.48 Q
Q_2	0.52 Q

V. A RESULTS FOR CENTRALISED APPROACH

DECISION VARIABLES	VALUES
Q^*	1328.7
$\in^*$	0.4414
${S_1}^*$	53.32
${S_2}^*$	45.72
PROFIT FUNCTION	VALUES
Manufacturer profit (π_M)	8789.03
Retailer R_1 profit (π_{R_1})	2643.61
Retailer R_2 profit (π_{R_2})	2208.48
Total profit considering all members$(\pi_M + \pi_{R_1} + \pi_{R_2})$	13641.12

Solutions are optimal as eigen-values of HC_1 are $-1800.05, -32.40, -9.54, -0.0013$.(all negative).

V. B RESULTS FOR DECENTRALISED APPROACH

DECISION VARIABLES	VALUES
Q^*	1549.19
$\in^*$	0.2666
${S_1}^*$	43.89
${S_2}^*$	38.34
PROFIT FUNCTION	VALUES
Manufacturer profit (π_M)	8834.81
Retailer R_1 profit (π_{R_1})	2580.29
Retailer R_2 profit (π_{R_2})	1515.59
Total profit considering all members$(\pi_M + \pi_{R_1} + \pi_{R_2})$	12930.69

$\left.\frac{\partial^2 \pi_{R_1}}{\partial {S_1}^2}\right|_{S_1 = {S_1}^*} = -18$, $\left.\frac{\partial^2 \pi_{R_2}}{\partial {S_2}^2}\right|_{S_2 = {S_2}^*} = -24$ and eigen values of the hessian matrix associated with manufacturer profit

doi: 10.32622/ijrat.103202204

$$\begin{bmatrix} \frac{\partial^2 \pi_P}{\partial Q^2} & \frac{\partial^2 \pi_P}{\partial Q\, \partial \in} \\ \frac{\partial^2 \pi_P}{\partial Q\, \partial \in} & \frac{\partial^2 \pi_P}{\partial \epsilon^2} \end{bmatrix} \text{ are } -1800, -0.000645497$$

All these above things confirm optimality of the solution.

This numerical section is developed with the help of MATHEMATICA SOFTWARE.

VI. CONCLUSION

In this work of mathematical modeling Green technology along with its effect has been incorporated. It also features with rivalry among retailers in terms of selling price. Proposed model is solved under two different approaches namely centralized and decentralized. From numerical study it reveals that total profit, considering all members profit, is much in centralized system rather than decentralized system. But manufacturer is more gainer in the decentralized approach which he leads.

There are many scopes for future extension of this model. This model can be extended for perishable goods. Different rates of deterioration for different members should be taken into consideration. Another interesting way of study if demand rate is taken as random in nature.

REFERENCES

[1] N. Absi, S.D-Peres, S.K-SIdhoum, B. Penz, C. Rapine, " Lot sizing with carbon emission constraints", European Journal of Operational Research, vol .227, pp. 55-61,2013.

[2] C.Y.Dye, C.T.Yang, " Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints" , European Journal of Operational Research, pp. 1-14, 2015.

[3] S. An, B.Li, D. Song, X. Chen, "Green credit financing versus trade credit financing with carbon emission limits" , European Journal of Operational Research, 2020,https://doi.org/10.1016/j.ejor.2020.10.025

[4] Y. Fang, Y. Yu, Y. Shi, J. Liu, " The effect of carbon tariff's on global emission control: A global supply chain model" , Transportation Research Part E, vol.133,101818,2020.

[5] Q. Bai, Y. Gong, M. Jin, X.Xu, " Effects of carbon emission reduction on supply chain co-ordination with vendor managed deteriorating product inventory", International Journal of Production Economics, 2018, https://doi.org/10.1016/j.ijpe.2018.11.008

[6] P. He, Z. Wang, V. Shi, Y. Liao, " The direct and cross effects in a supply chain with consumers sensitive to both carbon emissions and delivery time", European Journal of Operational Research,2020, https://doi.org/10.1016/j.ejor.2020.10.031

[7] J-Y Lee, S. Choi, " Supply chain investment and contracting for carbon emission reduction: A social planner's perspective", International Journal of Production Economics, vol.231, 107873, 2021.

[8] B. Sarkar, M. Sarkar, B. Ganguly, L. E. Cardenas Barron, " Combined effects of carbon emission and production quality improvement for fixed life time products in a sustainable supply chain management", International Journal of Production Economics, vol.231, 107867, 2021.

[9] A. Mandal, B. Pal, " Effects of green innovation and advertisement in an imperfect production- based competitive supply chain under two-tier credit facility", Mathematical Methods in Applied Sciences", vol 44,issue17, pp.13227-13251,2021.

[10] B. Pal, A. Sarkar, B. Sarkar, " Optimal decisions in a dual-channel competitive green supply chain management under promotional effort", Expert System with Applications, 211,118315,2023.

[11] V. Hovelaque, L. Bironneau, " The carbon-constrained EOQ model with carbon emission dependent demand, International Journal of ProductionEconomics, http://dx.doi.org/10.1016/j.ijpe.2014.11.022 ,2015.

[12] B. Sarkar, R. Guchhait, M. Sarkar, L.E. Cardenas Barron, " How does an industry manage cash flow within asmart production system with the carbon footprint and carbon emission under logistics frame work?" , International Journal of ProductionEconomics, vol.213,pp 243-257, 2019.

[13] S. Song, K. Govindan, L. Xu, P. Du, X. Qiao, " Capacity and production planning with carbon emission constraints, Transporation Research Part E, vol.97,pp.132-150, 2017.

[14] H-C Li, "Optimal delivery strategies considering carbon emissions, time-dependent demands and demand-supply interactions, European Journal of Operational Research, vol .241, pp. 739-748,2015.

AUTHORS PROFILE

Subhankar Adhikari is an Assistant Professor in Mathematics of Chandernagore College. He has obtained M.SC degree in Mathematics from Jadavpur University, Kolkata. And Ph.D degree in Mathematics from Burdwan University, Burdwan, India. He has published papers in International Journal of System Science: Operations and Logistics, RAIRO-Operations Research, International Journal of Mathematics in Operational Research and Journal of Industrial and Production Engineering. His research area includes inventory system and supply chain management.