International Journal of Research in Advent Technology, Vol.11, No.4, December 2023
E-ISSN: 2321-9637
Available online at www.ijrat.org

Numerical Solution of Product Type Fuzzy Volterra Integral Equation

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Abstract—An iterative algorithm is presented for approximating the solution of the product type fuzzy Volterra integral equation. Firstly, the uniqueness of the solution of the original integral equation is proved by using Banach fixed point theorem. Next, the error estimation of the proposed iterative method is achieved. Finally, two numerical examples are given to illustrate the effectiveness of the method.

Index Terms—fuzzy Volterra integral equation; product type; iterative method

I. INTRODUCTION

Integral equations are widely used in different fields, such as medicine, potential theory, mechanics and natural science [1-5]. As a special case, product type integral appears in the study of an infectious disease that does not cause permanent immunity [6-10].

In 1981, the existence and uniqueness of solutions for the following product type integral equations were studied by Gripenberg [9]

$$h(s) = k\left(p(s) + \int_0^s A(s-\eta)h(\eta)d\eta\right)\left(q(s) + \int_0^s B(s-\eta)h(\eta)d\eta\right), \quad (1)$$

where $h(s)$ is an unknown function and functions p, q are related to past infection.

In 1995, a new integral inequality was proposed by Pachpatte [11] to study the approximation of solutions of Eq. (1). Later Abdeldaim [12] and Li [13] further enriched the Pachpatte inequality.

In 2018, Boulfoul [14] studied the following product type integral equation

$$h(s) = f\big(s, h(s)\big) + f_1\left(\eta, \int_0^s v_1\big(s,\eta,h(\eta)\big)d\eta\right) f_2\left(\eta, \int_0^s v_2\big(s,\eta,h(\eta)\big)d\eta\right), \quad (2)$$

where $h(s)$ is an unknown function, $f\big(s,h(s)\big)$ is a fuzzy source function and f_1, f_2 obey linear growth in the second independent variable.

It is well known that the parameters in the integral equation are often uncertain in real mathematical modeling. The concept of fuzzy was proposed and have been effectively developed.

In 2021, Mahaloh [15] proposed the uniqueness of the solution of the following product type fuzzy Fredholm integral equation

$$h(s) = f(s) \oplus (FR)\int_a^b k_1\big(s,\eta,h(\eta)\big)d\eta \odot (FR)\int_a^b k_2\big(s,\eta,h(\eta)\big)d\eta, s \in [a,b], \quad (3)$$

where $h(s)$ is an unknown function, $f(s)$ is a fuzzy source function and $(FR)\int_a^b * \, d\eta$ denotes the Riemann integrable function on $[a,b]$.

In this paper, we study the following product type fuzzy Volterra integral equations

$$h(s) = g(s) + \int_0^s B_1\big(s,\eta,h(\eta)\big)d\eta \int_0^s B_2\big(s,\eta,h(\eta)\big)d\eta, s \in [0,1], \quad (4)$$

where $h(s)$ is an unknown function and $g(s)$ is a fuzzy source function.

However, there are few studies on solving Eq.(4). We regard the set of one-dimensional fuzzy numbers as a closed convex dimension in a Banach space, and prove the uniqueness of the solution of Eq.(4) by using Banach fixed point theorem. The error estimation of the iterative algorithm is analyzed.

The iteration algorithm of Eq.(4) is proposed by

$$\begin{aligned} w_0 &= g(s), \\ w_m &= g(s) + \int_0^s B_1(s,\eta,v_{m-1})d\eta \int_0^s B_2(s,\eta,v_{m-1})d\eta. \end{aligned} \quad (5)$$

The structure is as follows: Section 2 introduces some fuzzy concepts. In Section 3, proves the uniqueness of the solution of the original integral equation. Section 4 gives the error analysis of the iterative method. In Section 5, two numerical examples are given to illustrate the effectiveness of the proposed method. Finally, a brief summary is made in Section 6.

II. PRELIMINARIES

In this section, some basic concepts in fuzzy calculus are recalled, as given in [16-19].

Definition2.1. [17, 18] $\mathcal{F}(\mathrm{R})$ denotes the set of all fuzzy sets on R. Let $\mathrm{h} \in \mathcal{F}(\mathrm{R})$, if h satisfies

(i) h is a normal fuzzy set, i.e., there exists $\mathrm{s_0} \in \mathrm{R}$ such that $\mathrm{h(s_0)} = 1$,

(ii) h is a convex fuzzy set, i.e., $\mathrm{h(\delta s_1 + (1-\delta)s_2)} \geq \min\{\mathrm{h(s_1), h(s_2)}\}$ for all $\mathrm{s_1, s_2} \in \mathrm{R}$ and $\delta \in [0,1]$,

(iii) h is an upper semi-continuous function,

The manuscript was revised on September 17, 2023 and published on October 17, 2023
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doi: 10.32622/ijrat.112202301

(iv) The closure of the support of h is compact, i.e., $[h]^0$ is compact,

then h is called as a fuzzy number. The set of all fuzzy numbers is known as the fuzzy number space, denoted by E.

Definition2.2. [19] Given $0 \le r \le 1$, a fuzzy number h in parametric form is represented by an ordered function pairs $\left(\underline{h}(r), \overline{h}(r)\right)$ satisfying

(i) $\underline{h}(r)$ is a bounded left continuous non decreasing function,

(ii) $\overline{h}(r)$ is a bounded left continuous non increasing function,

(iii) $\underline{h}(r) \le \overline{h}(r)$.

For $h = (\underline{h}, \overline{h}), v = (\underline{v}, \overline{v}) \in E$ and $\delta \in R$, the sum of $v + h$ and the scalar multiplication δh can be defined by

$$\left(\underline{v+h}\right)(r) = \underline{v}(r) + \underline{h}(r), \quad \left(\overline{v+h}\right)(r) = \overline{v}(r) + \overline{h}(r), \quad \forall r \in [0,1],$$

and

$$\delta h = \begin{cases} (\delta\underline{h}, \delta\overline{h}), \delta \ge 0, \\ (\delta\overline{h}, \delta\underline{h}), \delta \le 0. \end{cases}$$

Definition2.3. [20]For any two fuzzy numbers w and h, define $D_r: E \times E \to R^+ \cup \{0\}$by

$$D_r(v, h) = \sup_{r \in [0,1]} \max\{|\underline{v}(r) - \underline{h}(r)|, |\overline{v}(r) - \overline{h}(r)|\}.$$

where $v = [\underline{v}(r), \overline{v}(r)], h = [\underline{h}(r), \overline{h}(r)]$. It has the following useful propertie.

For $\forall w, h, v, \alpha \in E$, there are

(i) (E, D_r) is a complete metric space,

(ii) $D_r(w + v, h + v) = D_r(w, h)$,

(iii) $D_r(w, h) \le D_r(w, v) + D(v, h)$,

(iv) $D_r(\alpha w, \alpha h) = \| \alpha \| D_r(w, h)$, (see [23]).

(v) $\| w \| = D_r(w, \tilde{0})$, (see [24]).

(vi) $D_r\left(\int_J w(s)ds, \int_J h(s)ds\right) \le \int_J D_r\left(w(s), h(s)\right)ds$,

(vii) $D_r(w \tilde{*} h, \tilde{0}) = D_r(w, \tilde{0}) D_r(h, \tilde{0})$ with the fuzzy multiplication $\tilde{*}$ is based on the extension principle that can be proved by α -cuts of fuzzy numbers $w, h \in E$. Here $\tilde{0} \in E$ is defined by (see [25])

$$\tilde{0}(t) = \begin{cases} 1, s = 0, \\ 0, elsewhere. \end{cases}$$

III. UNIQUENESS RESULT

The contribution of this section is devoted to the uniqueness of the solution of Eq (4). It can be achieved by using Banach fixed point theorem based on the following hypothesis.

(A1) $g \in C_F([0,1], E)$.

(A2) There exist $\mu_1 > 0$ and $\mu_2 > 0$ such that

$$D_r(B_1(s, \eta, h(\eta), \tilde{0}) \le \mu_1, D_r(B_2(s, \eta, h(\eta), \tilde{0}) \le \mu_2.$$

(A3) There are two numbers $L_1 > 0, L_2 > 0$ such that

$$D_r(B_1(s, \eta, h_1(\eta), B_1(s, \eta, h_2(\eta)) \le L_1 D_r(h_1, h_2),$$
$$D_r(B_2(s, \eta, h_1(\eta), B_2(s, \eta, h_2(\eta)) \le L_2 D_r(h_1, h_2),$$

where $0 < C^* := L_1\mu_2 + L_2\mu_1 < 1$.

Theorem 1. Assume that (A1)-(A3) hold, then Eq (4) has an unique solution.

Proof. For $\forall h_1, h_2 \in E$. Define the following operator $P: C_F([0,1], E) \to C_F([0,1], E)$

$$(Ph)(s) = \quad g(s) + \int_0^s B_1\left(s, \eta, h(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h(\eta)\right)d\eta.$$

We can get

$$\begin{aligned}
& D_r\left(P(h_1(s)), P(h_2(s))\right) \\
= \; & D_r(g(s) + \int_0^s B_1\left(s, \eta, h_1(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_1(\eta)\right)d\eta, \\
& g(s) + \int_0^s B_1\left(s, \eta, h_2(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_2(\eta)\right)d\eta) \\
\le \; & D_r(\int_0^s B_1\left(s, \eta, h_1(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_1(\eta)\right)d\eta, \\
& \int_0^s B_1\left(s, \eta, h_1(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_2(\eta)\right)d\eta) \\
+ \; & D_r(\int_0^s B_1\left(s, \eta, h_1(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_2(\eta)\right)d\eta, \\
& \int_0^s B_1\left(s, \eta, h_2(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h_2(\eta)\right)d\eta) \\
\le \; & \int_0^s D_r\left(B_1(s, \eta, h_1(\eta)), \tilde{0}\right)d\eta \int_0^s D_r\left(B_2(s, \eta, h_1(\eta)),\right. \\
& \left.B_2(s, \eta, h_2(\eta))\right)d\eta + \int_0^s D_r\left(B_2(s, \eta, h_1(\eta)), \tilde{0}\right)d\eta \\
& \int_0^s D_r\left(B_1(s, \eta, h_1(\eta)), B_1(s, \eta, h_2(\eta))\right)d\eta \\
\le \; & (\mu_1 L_2 + \mu_2 L_1) D_r(h_1, h_2) := C^* D_r(h_1, h_2).
\end{aligned}$$

Shows that P is a contraction map. Eq.(4) has an unique solution h^* by using Banach fixed point theorem. □

IV. ERROR ESTIMATION

Let h^* and w_m be the exact solution of Eq.(4) and the approximate solution of Eq.(5).

Theorem 2. Under the conditions (A1)-(A3), then the following error estimation holds

$$D_r\left(h^*(s), w_m(s)\right) \le \frac{\mu^2 C^{*m}}{1 - C^*}, \textit{where } \mu = max\{\mu_1, \mu_2\}.$$

Proof. By using Definition 2.3 and the triangle inequality, there is

$$\begin{aligned}
& D_r\left(h^*(s), w_m(s)\right) \\
= \; & D_r(g(s) + \int_0^s B_1\left(s, \eta, h^*(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h^*(\eta)\right)d\eta, \\
& g(s) + \int_0^s B_1\left(s, \eta, w_{m-1}(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, w_{m-1}(\eta)\right)d\eta) \\
\le \; & D_r(\int_0^s B_1\left(s, \eta, h^*(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, h^*(\eta)\right)d\eta, \\
& \int_0^s B_1\left(s, \eta, h^*(\eta)\right)d\eta \int_0^t B_2\left(s, \eta, w_{m-1}(\eta)\right)d\eta) \\
+ \; & D_r(\int_0^s B_1\left(s, \eta, h^*(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, w_{m-1}(\eta)\right)d\eta, \\
& \int_0^s B_1\left(s, \eta, w_{m-1}(\eta)\right)d\eta \int_0^s B_2\left(s, \eta, w_{m-1}(\eta)\right)d\eta) \\
\le \; & \int_0^s D_r\left(B_1(s, \eta, h^*(\eta)), \tilde{0}\right)d\eta \int_0^s D_r\left(B_2(s, \eta, h^*(\eta)),\right.
\end{aligned}$$

doi: 10.32622/ijrat.112202301

$B_2(s,\eta,w_{m-1}(\eta)))\mathrm{d}\eta + \int_0^s D_r\left(B_2(s,\eta,w_{m-1}(\eta)),\tilde{0}\right)\mathrm{d}\eta$

$\int_0^s D_r\left(B_1(s,\eta,h^*(\eta)),B_1(s,\eta,w_{m-1}(\eta))\right)\mathrm{d}\eta$

$\leq \mu_1 L_2 D_r(h^*,w_{m-1}) + \mu_2 L_1 D_r(h^*,w_{m-1})$

$\leq C^* D_r(h^*,w_{m-1})$ (6)

Combining

$$D_r(h^*,w_{m-1}) \leq D_r(h^*,w_m) + D_r(w_m,w_{m-1}),$$

and Eq.(4.1), there is

$$D_r(h^*,w_m) \leq \frac{C^*}{1-C^*} D_r(w_m,w_{m-1}). \quad (7)$$

By using Definition 2.3, we have

$D_r\left(w_m(s),w_{m-1}(s)\right)$

$= D_r(g(s) + \int_0^s B_1\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta,$

$g(s) + \int_0^s B_1\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta)$

$\leq D_r(\int_0^s B_1\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta,$

$\int_0^s B_1\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta)$

$+ D_r(\int_0^s B_1\left(s,\eta,w_{m-1}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta,$

$\int_0^s B_1\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,w_{m-2}(\eta)\right)\mathrm{d}\eta)$

$\leq \int_0^s D_r\left(B_1(s,\eta,w_{m-1}(\eta)),\tilde{0}\right)\mathrm{d}\eta \int_0^s D_r\left(B_2(s,\eta,w_{m-1}(\eta)),\right.$

$B_2(s,\eta,w_{m-2}(\eta)))\mathrm{d}\eta + \int_0^s D_r\left(B_2(s,\eta,w_{m-2}(\eta)),\tilde{0}\right)\mathrm{d}\eta$

$\int_0^s D_r\left(B_1(s,\eta,w_{m-1}(\eta)),B_1(s,\eta,w_{m-2}(\eta))\right)\mathrm{d}\eta$

$\leq C^* D_r(w_{m-1},w_{m-2}).$

Hence

$$D_r(w_m,w_{m-1}) \leq C^* D_r(w_{m-1},w_{m-2}) \cdots \leq C^{*m-1} D_r(w_1,w_0). \quad (8)$$

For $w_0 = g(s)$, there is

$D_r\left(w_1(s),w_0(s)\right)$

$= D_r\left(\int_0^s B_1\left(s,\eta,g(\eta)\right)\mathrm{d}\eta \int_0^s B_2\left(s,\eta,g(\eta)\right)\mathrm{d}\eta, g(s)\right)$

$\leq \int_0^s D_r\left(B_1(s,\eta,g(\eta)),\tilde{0}\right)\mathrm{d}\eta \int_0^s D_r\left(B_2(s,\eta,g(\eta)),\tilde{0}\right)$

$\leq \mu_1\mu_2 := \mu^2$ (9)

where $\mu = max\{\mu_1,\mu_2\}$.

From Eqs.(6)-(9), it is clear that

$$D_r\left(h^*(s),w_m(s)\right) \leq \frac{C^{*m}\mu^2}{1-C^*}.$$

V. NUMERICAL EXAMPLES

Example 1 Consider the following product type fuzzy Volterra integral equation

$$h(s) = g(s) + \int_0^s B_1\left(s,\eta,h(\eta)\right)d\eta \int_0^s B_2\left(s,\eta,h(\eta)\right)d\eta,$$

where

$$g(s) = \left(\underline{g}(s,r),\overline{g}(s,r)\right) = \left(rx - \frac{r^2}{18}s^6, (2-r)s - \frac{(2-r)^2}{18}s^6\right), s,r \in [0,1],$$

and kernels

$$B_1\left(s,\eta,w(\eta,r)\right) = \frac{1}{3}\eta h(\eta,r), B_2\left(s,\eta,w(\eta,r)\right) = sh(\eta,r).$$

The exact solution is $h(s) = \left(\underline{h}(s,r),\overline{h}(s,r)\right) = (rs,(2-r)s)$.

Taking $r = 0.4$ and $n = 5$, the error are given in Table 1 and Table 2

Table 1 *Left bound of errors (when* $r = 0.4, n = 5$*)*

s	Exact	Approximation	E_L
0	0	0	0
0.2	8.0000e-02	8.0000e-02	1.3878e-17
0.4	1.6000e-01	1.6000e-01	2.7756e-17
0.6	2.4000e-01	2.4000e-01	5.5511e-17
0.8	3.2000e-01	3.2000e-01	1.0825e-13
1.0	4.0000e-01	4.0000e-01	3.5762e-11

Table 2 *Right bound of errors (when* $r = 0.4, n = 5$*)*

s	Exact	Approximation	E_R
0	0	0	0
0.2	3.2000e-01	3.2000e-01	5.5511e-17
0.4	6.4000e-01	6.4000e-01	1.1102e-16
0.6	9.6000e-01	9.6000e-01	2.5013e-13
0.8	1.2800e+00	1.2800e+00	4.4248e-10
1.0	1.6000e+00	1.6000e+00	1.4573e-07

Example 2 Consider the following product type fuzzy Volterra integral equation

$$h(s) = g(s) + \int_0^s B_1\left(s,\eta,h(\eta)\right)d\eta \int_0^s B_2\left(s,\eta,h(\eta)\right)d\eta,$$

where

$$g(s) = \left(\underline{g}(s,r),\overline{g}(s,r)\right) = \left(\frac{r}{2}s^2 - \frac{r^2}{8}s^{10}, \frac{2-r}{2}s^2 - \frac{(2-r)^2}{8}s^{10}\right), s,r \in [0,1]$$

and kernels

$$B_1\left(s,\eta,h(\eta,r)\right) = 2s^2 h(\eta,r), B_2\left(s,\eta,h(\eta,r)\right) = 3s\eta h(\eta,r).$$

The exact solution is $h(s) = \left(\underline{h}(s,r),\overline{h}(s,r)\right) = \left(\frac{r}{2}s^2, \frac{2-r}{2}s^2\right)$. Taking $r = 0.5$ and $n = 5$, the errors are shown in Table 3 and Table 4.

Table 3 *Left bound of errors (when* $r = 0.5, n = 5$*)*

s	Exact	Approximation	E_L
0	0	0	0

doi: 10.32622/ijrat.112202301

s	Exact	Approximation	E_L
0.2	3.0000e-02	3.0000e-02	6.9389e-18
0.4	1.2000e-01	1.2000e-01	2.7756e-17
0.6	2.7000e-01	2.7000e-01	2.9032e-14
0.8	4.8000e-01	4.8000e-01	5.1079e-09
1.0	7.5000e-01	7.4994e-01	5.8568e-05

Table 4 *Right bound of errors (when $r = 0.5, n = 5$)*

s	Exact	Approximation	E_R
0	0	0	0
0.2	1.0000e-02	1.0000e-02	1.7347e-18
0.4	4.0000e-02	4.0000e-02	6.9389e-18
0.6	9.0000e-02	9.0000e-02	4.1633e-17
0.8	1.6000e-01	1.6000e-01	7.0272e-12
1.0	2.5000e-01	2.5000e-01	8.1995e-08

According to the results of numerical examples in Table 1-Table 4, it can be seen that the error between the approximate solution and the exact solution of the left and right boundaries of the product type fuzzy Volterra integral equation solved by the iterative algorithm is very small, and the effect is ideal. It also shows the applicability and effectiveness of using the iterative method to solve this kind of equation.

VI. CONCLUSION

In this paper, we consider the numerical solution of product type fuzzy Volterra integral equation. The first result is to prove the uniqueness of the solution of the original product type fuzzy Volterra integral equation. The second result is the error estimation of the iterative algorithm. Numerical results show that the iterative algorithm is effective. In the following work, we will consider extending the method to the nonlinear case and study its numerical solution

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doi: 10.32622/ijrat.112202301

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doi: 10.32622/ijrat.112202301