International Journal of Research in Advent Technology, Vol.7, No.3, March 2019
E-ISSN: 2321-9637
Available online at www.ijrat.org

New Class of Generalized Closed Sets in Supra Topological Spaces

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Abstract- In this paper we introduce supra $\tilde{\tilde{g}}_\alpha$-closed sets and we show that the class of supra $\tilde{\tilde{g}}_\alpha$-closed sets lies between the class of supra α-closed sets and the class of supra generalized semi-closed sets. We also introduce and discuss some properties of $T^{\mu}_{\tilde{\tilde{g}}_\alpha}$-spaces, $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}\alpha}}$-spaces, $\#_{T^{\mu}_{\tilde{\tilde{g}}\alpha}}$-spaces. Further we define and investigate the properties of supra-$\tilde{\tilde{g}}_\alpha$-continuous functions.

Index Terms- Supra α-closed sets, Supra gs-closed sets, Supra $\tilde{\tilde{g}}_\alpha$-closed sets, Supra-$\tilde{\tilde{g}}_\alpha$-continuous function.

1. INTRODUCTION

Extensive research on generalizing closedness was done in recent years by many Mathematicians. In 1983, A.S.Mashhour et al.[17] introduced the concept of supra topological spaces. M.Caldas et al.[5] introduced and studied supra α-open sets and its continuity. In 2010, O.R.Sayed [20] concentrated on supra pre-open sets and supra pre-continuity on topological spaces. In 1970, Levine.N [13] introduced and established the properties of generalized closed sets in classical topology. Many researchers [1, 2, 4, 6, 18] turned their attention to define new sets and functions in topological spaces. In 1980, Levine and Dunham [7] further characterized some more properties of generalized closed sets. Noiri et al. [15] proved that every topological space is pre-$T_{\frac{1}{2}}$-space. Further some spaces are derived by Maheshwari and Prasad [16]. Recently Jayaparthasarathy, Hydar Akca and Jamal Benbourenane [8, 9] defined and investigated some properties of the new class of generalized closed sets namely $\tilde{\tilde{g}}_\alpha$-closed sets. In this paper we define and discuss a new class of weakly α- generalized closed set in supra topological spaces called supra $\tilde{\tilde{g}}_\alpha$-closed set and investigate its properties. We observe that the class of supra $\tilde{\tilde{g}}_\alpha$-closed sets lies between the class of supra α-closed sets and the class of supra generalized semi-closed sets. By applying supra $\tilde{\tilde{g}}_\alpha$-closed sets, we introduce and establish some properties of $T^{\mu}_{\tilde{\tilde{g}}_\alpha}$-spaces, $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}\alpha}}$-spaces. Moreover the supra $\tilde{\tilde{g}}_\alpha$-continuous functions are defined and its properties are investigated.

2. PRELIMINARIES

In this section we recall some basic definitions and properties of supra topology which are useful in sequel.

Definition 2.1 Let X be a non-empty set. A sub collection μ of P(X) where P(X) denote the power set of X, is said to be a supra topology on X [17] if

(i) $\emptyset$, X ∈ μ
(ii) μ is closed under arbitrary union.

The pair (X, μ) is called supra topological space. The element of μ are called the supra open sets in (X, μ) and the complement of the supra open sets are called supra closed sets.

Definition 2.2 Let (X, μ) be a supra topological space and A ⊆ X .Then

(i) The supra closure of A is denoted by cl^{μ}(A)[17], defined as cl^{μ} (A)=∩ {B : B is supra closed set and A ⊆ B}.
(ii) The supra interior of A is denoted by int^{μ} (A) [17], defined as int^{μ} (A)=∪ {B : B is supra open set and A ⊇ B}.

Definition 2.3 Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ [17] if τ ⊆ μ.

Definition 2.4 Let (X , μ) be a supra topological space. A subset A of X is called

i) supra α-open [14] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.
ii) supra semi-open [12] if $A \subseteq cl^{\mu}(int^{\mu}(A))$.
iii) supra pre-open [3] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.

The complement of a supra α-open set (resp. supra semi-open, and supra pre-open) is called supra α-closed (resp. supra semi-closed, and supra pre-closed).

Theorem 2.5 (i) Every supra open set (supra closed set) is supra α-open set (supra α-closed set) [5].
(ii) Let (X, τ) be a topological space and μ be a supra topology associated with τ . Then a subset A of X is supra α-open (supra α-closed) set if and only if it is supra semi-open (supra semi -closed) set and supra pre-open (supra pre-closed) set [5, 20].

Definition 2.6 A subset A of a supra topological space (X, μ) is called

(i) supra generalized-closed (briefly g^{μ}-closed) set [19] if cl^{μ} (A) ⊆ U whenever A ⊆ U and U is supra open set in (X, μ).
(ii) supra semi generalized-closed (briefly sg^{μ}-closed) set [19] if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is a supra semi-open set in (X, μ).
(iii) supra generalized semi-closed (briefly gs^{μ}-closed) set [19] if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open set in (X, μ).
(iv) supra α-generalized closed (briefly αg^{μ}-closed) set [19] if αcl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open set in (X, μ).
(v) supra generalized α-closed (briefly $g\alpha^{\mu}$-closed) set [19] if αcl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra α-open set in (X, μ).
(vi) supra $\hat{g}$-closed (briefly $\hat{g}^{\mu}$-closed) set [11] if αcl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra semi-open set in (X, μ).
(vii) supra $*_g$-closed (briefly ${*_g}^{\mu}$-closed) set [11] if cl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra $\hat{g}$-open set in (X, μ).
(viii) supra $\#_g$-semi-closed (briefly ${\#_{gs}}^{\mu}$-closed) set [11] if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra $*_g$-open set in (X, μ).
(ix) supra $\tilde{g}$-closed (briefly $\tilde{g}^{\mu}$-closed) set [11] if cl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra $\#_g$-semi-open set in (X, μ).
(x) supra $\tilde{g}$-semi-closed (briefly $\tilde{g}s^{\mu}$-closed) set [11] if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra $\#_g$-semi-open set in (X, μ).
(xi) supra $\tilde{\tilde{g}}$-semi-closed (briefly $\tilde{\tilde{g}}s^{\mu}$-closed) set [11] if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra $\tilde{g}$-semi-open set in (X, μ).

The complement of the above mentioned sets are called their respective generalized open sets.

Definition 2.7 [10] Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f: (X, τ) → (Y, σ) is called a

i) supra $\hat{g}$-continuous (briefly $\hat{g}^{\mu}$-continuous) map if the inverse image of each closed set in Y is a $\hat{g}^{\mu}$-closed set in X.
ii) supra $*_g$-continuous (briefly ${*_g}^{\mu}$-continuous) map if the inverse image of each closed set in Y is a ${*_g}^{\mu}$-closed set in X.
iii) supra $\#_g$-semi-continuous (briefly ${\#_{gs}}^{\mu}$-continuous) map if the inverse image of each closed set in Y is a ${\#_{gs}}^{\mu}$-closed set in X.
iv) supra $\tilde{g}$-continuous (briefly $\tilde{g}^{\mu}$-continuous) map if the inverse image of each closed set in Y is a $\tilde{g}^{\mu}$-closed set in X.
v) supra $\tilde{g}$-semi-continuous (briefly $\tilde{g}s^{\mu}$-continuous) map if the inverse image of each closed set in Y is a $\tilde{g}s^{\mu}$-closed set in X.
vi) supra $\tilde{\tilde{g}}$-semi-continuous (briefly $\tilde{\tilde{g}}s^{\mu}$-continuous) map if the inverse image of every closed set in (Y, σ) is $\tilde{\tilde{g}}s^{\mu}$--closed in X.

3. NEW CLASS OF SUPRA $\tilde{\tilde{g}}_{\alpha}$-CLOSED SETS

In this section we define and discuss a new class of weakly generalized closed sets in supra topological spaces called supra $\tilde{\tilde{g}}_{\alpha}$-closed sets and investigate its properties.

Definition 3.1 A subset A of X is called supra $\tilde{\tilde{g}}_{\alpha}$-closed set (briefly ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed) if αcl^{μ}(A) ⊆ U whenever A ⊆ U and U is $\tilde{g}s^{\mu}$-open set in (X, μ).

Theorem 3.2 Every supra closed set is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set.

Proof. If A is any supra closed set in X and U is any $\tilde{g}s^{\mu}$-open set containing A. Then αcl^{μ}(A) ⊆ cl^{μ}(A) = A ⊆ U. Hence A is a ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set.

Remark 3.3 The converse of the above theorem need not be true and it is shown by the following example.

Example 3.4 Let X = {a, b, c, d}, μ = {∅, {a}, {b}, {a, b}, X}, then the set {a} is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed but it is not supra closed.

Theorem 3.5 Every supra α-closed set is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set.

Proof. Let A be a supra α-closed and U be any $\tilde{g}s^{\mu}$-open set containing A, then αcl^{μ}(A) =A ⊆ U. Hence A is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed.

Remark 3.6 The converse of the above theorem need not be true as shown in the following example.

Example 3.7 Let X = {a, b, c, d}, μ = {∅, {a, b}, X}, then the set {a, c, d} is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed but not supra α-closed.

Theorem 3.8 Every $\tilde{g}^{\mu}$-closed set is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set but not conversely.

Proof. Let A be any $\tilde{g}^{\mu}$-closed set and U be any $\tilde{g}s^{\mu}$-open set containing A. Since every $\tilde{g}s^{\mu}$-open set is ${\#_{gs}}^{\mu}$-open [11], we have A ⊆ U and αcl^{μ} (A) ⊆ cl^{μ}(A) ⊆ U. Hence A is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed.

Example 3.9 Let X = {a, b, c, d}, μ = {∅, {a, b}, X}. Then the set {c} is ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed but not $\tilde{g}^{\mu}$-closed.

Theorem 3.10 Every ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set is gs^{μ}-closed .

Proof. Let A be a ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set and U be any supra open set containing A. Since any supra open set is $\tilde{g}s^{\mu}$-open, we have αcl^{μ}(A) ⊆ U. Hence A is gs^{μ}-closed.

Remark 3.11 The converse of the above theorem need not be true as shown in the following example.

Example 3.12 Let X = {a, b, c}, μ = {∅, {b}, X}. Then the set {a, b} is gs^{μ}-closed but not ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed.

Theorem 3.13 Every ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set is sg^{μ}-closed but not conversely.

Proof. Let A be a ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set and U be any supra semi-open set containing A. Since any supra semi-open set is $\tilde{g}s^{\mu}$-open, we have αcl^{μ}(A) ⊆ U. Hence A is sg^{μ}-closed.

Example 3.14 Let X = {a, b, c, d}, μ = {∅, {a}, {b, c}, {a, b, c}, X}. Then the set {c} is sg-closed but not ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed.

Proposition 3.15 Every ${\tilde{\tilde{g}}_{\alpha}}^{\mu}$-closed set is $\tilde{\tilde{g}}s^{\mu}$-closed but not conversely.

Proof: Let A be a $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set and U be any $\tilde{g}s^{\mu}$-closed set containing A, we have scl^{μ} (A) ⊆ αcl^{μ}(A) ⊆ U. Hence A is $\tilde{\tilde{g}}s^{\mu}$-closed.

Example 3.16 Let X = {a, b, c, d}, μ = {∅, {a}, {b, c}, {a, b, c}, X}. Then the set {b, c} is $\tilde{\tilde{g}}s^{\mu}$-closed set but not $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

Remark 3.17 Here we observe that the class of $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed sets is properly placed between the class of supra α-closed sets and the class of gs^{μ}-closed sets and between the class of supra α-closed sets and the class of $\tilde{\tilde{g}}s^{\mu}$-closed sets.

Remark 3.18 The following examples shows that $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set is independent from supra- semi-closed set, g^{μ}-closed set.

Example 3.19 (i) Let X = {a, b, c}, μ = {∅, {a}, X}. Then the set {a, b} is supra-g-closed but not $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

(ii) Let X = {a, b, c}, μ = {∅, {a}, {a, b}, X}. Then the set {b} is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed but not supra-g-closed.

(iii) Let X = {a, b, c}, μ = {∅, {a}, {b}, {a, b}, X}. Then the set {a} is supra-semi-closed, but not $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

(iv) Let X = {a, b, c, d}, μ = {∅, {a}, {b, c}, {a, b, c}, X}. Then the set {b, d} is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed but not supra-semi-closed .

Definition 3.20 A subset A of X is called $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open if and only if A^c is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

Theorem 3.21 (i) Every supra-open set is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open but not conversely.

(ii) Every supra-α-open set is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open but not conversely.

(iii) Every $\tilde{g}^{\mu}$-open set is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open but not conversely.

(iv) Every $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set is supra-gs-open, supra-sg-open but not conversely.

(v) Every $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set is supra-αg-open, supra-gα-open but not conversely.

(vi) Every $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set is $\tilde{\tilde{g}}s^{\mu}$-open set but not conversely.

4. CHARACTERIZATION OF SUPRA $\tilde{\tilde{g}}_{\alpha}$-CLOSED SETS

In this section by applying $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed sets, some new classes of spaces and its properties are investigated.

Theorem 4.1 A set A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set of (X, μ), then αcl^{μ}(A)−A does not contain any non empty $\tilde{g}s^{\mu}$-closed set.

Proof. Suppose that A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed in (X, μ). Let F be a $\tilde{g}s^{\mu}$-closed subset of αcl^{μ}(A)−A. Now F^c is $\tilde{g}s^{\mu}$-open set of (X, μ) such that A ⊆ F^c. Since A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set, αcl^{μ}(A) ⊆ F^c, that is, F ⊆ $(\alpha cl^{\mu}(A))^c$. Thus F ⊆ αcl^{μ}(A) ∩ $(\alpha cl^{\mu}(A))^c$ = ϕ. Hence F is empty.

Theorem 4.2 If A is $\tilde{g}s^{\mu}$-open and $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed subset of (X, μ), then A is supra α-closed.

Proof. Since A is $\tilde{g}s^{\mu}$-open and $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed, αcl^{μ}(A) ⊆ A. But A ⊆ αcl^{μ}(A). Thus αcl^{μ}(A) = A. hence A is supra α-closed.

Theorem 4.3 If A is a $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set in supra topological space (X, μ) and A ⊆ B ⊆ αcl^{μ}(A) then B is also $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

Proof. Let U be any $\tilde{g}s^{\mu}$-open set such that B ⊆ U. Since A ⊆ B implies A ⊆ U. Since A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed then αcl^{μ}(A) ⊆ U. Since B ⊆ αcl^{μ}(A), αcl^{μ}(B) ⊆ αcl^{μ}(A) ⊆ U. Thus αcl^{μ}(B) ⊆ U and hence B is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

Theorem 4.4 Let A be $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set in supra topological space (X, μ), then A is supra α-closed subset of (X, μ) if and only if αcl^{μ}(A) − A is $\tilde{g}s^{\mu}$-closed.

Proof. Let A be $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set. If A is supra α-closed subset of (X, μ). Then αcl^{μ}(A) = A and so αcl^{μ}(A) − A = ∅ which is $\tilde{g}s^{\mu}$-closed. Conversely, let αcl^{μ}(A) − A is $\tilde{g}s^{\mu}$-closed set. Since A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed, by Theorem 4.1, αcl^{μ}(A) − A does not contain any non-empty $\tilde{g}s^{\mu}$-closed set, then αcl^{μ}(A) − A = ∅. That is αcl^{μ}(A) = A. Hence A is supra α-closed set in (X, μ).

Theorem 4.5 The intersection of a $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set and a supra α-closed set is always $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set in (X, μ).

Proof. Let A be $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set and let F be supra α-closed set. If G is an $\tilde{g}s^{\mu}$-open set with A ∩ F ⊆ G, then A ⊆ G∪F^c. Then αcl^{μ}(A ∩ F) ⊆ αcl^{μ}(A) ∩ F ⊆ G. Hence A ∩ F is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed.

Theorem 4.6 A set A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open in (X, μ) if and only if F ⊆ αint^{μ}(A) whenever F is $\tilde{g}s^{\mu}$-closed and F ⊆ A.

Proof. Let A be $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set and suppose F ⊆ A where F is $\tilde{g}s^{\mu}$-closed set. Then X - A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed set contained in the $\tilde{g}s^{\mu}$-open set X - F. Hence αcl^{μ}(X − A) ⊂ X − F. Thus F ⊂ αcl^{μ}(A). Conversely, if F is a $\tilde{g}s^{\mu}$-closed with F ⊆ αint^{μ}(A) and F ⊆ A, then X − αint^{μ}(A) ⊂ X − F. This implies that αcl^{μ}(X − A) ⊂ X − F. Hence X - A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed. Thus A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set in (X, μ).

Theorem 4.7 If αint^{μ}(A) ⊆ B ⊆ A and A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open then B is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open.

Proof. By hypothesis $(A)^c$ ⊆ $(B)^c$ ⊆ $(\alpha int^{\mu}(A))^c$. That is, $(A)^c$ ⊆ $(B)^c$ ⊆ $(X − \alpha cl^{\mu}(A^c))^c$ = $\alpha cl^{\mu}(A^c)$. Since A^c is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed and by Theorem 4.3, B^c is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed. Hence B is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open.

Theorem 4.8 A set A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open in (X, τ) if and only if G = X, where G is $\tilde{g}s^{\mu}$-open set and αint^{μ}(A) ∪ A^c ⊆ G.

Proof. Let A be a $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open set and G be $\tilde{g}s^{\mu}$-open and αint^{μ}(A) ∪ A^c ⊆ G. This gives G^c ⊆ $(\alpha int^{\mu}(A) \cup A^c)^c$ = $\alpha cl^{\mu}(A^c)$ ∩ A = $\alpha cl^{\mu}(A^c)$ − $(A)^c$. Since A^c is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-closed and G^c is $\tilde{g}s^{\mu}$-closed. Then G^c = ∅. Thus G = X. Conversely, if F is $\tilde{g}s^{\mu}$-closed and F ⊆ A. Then αint^{μ}(A) ∪ A^c ⊆ αint^{μ}(A) ∪ F^c. It follows by hypothesis that αint^{μ}(A) ∪ F^c = X and hence F ⊆ αint^{μ}(A). Therefore by Theorem 4.6, A is $\tilde{\tilde{g}}_{\alpha}^{\mu}$-open in (X, μ).

Theorem 4.9 For each x ∈ X either {x} is $\tilde{g}s^{\mu}$-closed or $\{x\}^c$ is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set in(X, μ).

Proof. If {x} is not $\tilde{g}s^{\mu}$-closed set in (X, μ), then $\{x\}^c$ is not $\tilde{g}s^{\mu}$-open and the only $\tilde{g}s^{\mu}$-open set containing $\{x\}^c$ is the space X itself. Therefore $\alpha cl^{\mu}(\{x\}^c) \subseteq X$ and so $\{x\}^c$ is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set in (X, μ).

Definition 4.10 A space (X, μ) is called a $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space if every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set is α-closed.

Theorem 4.11 For a supra topological space (X, μ), the following are equivalent.

(i) X is a $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space.

(ii) Every singleton of X is either $\tilde{g}s^{\mu}$-closed or supra α-open.

Proof. (i) ⇒ (ii): Let x ∈ X. Suppose {x} is not $\tilde{g}s^{\mu}$-closed, then X − {x} is not $\tilde{g}s^{\mu}$-open. Since X is the only $\tilde{g}s^{\mu}$-open set containing X−{x}. Thus X−{x} is $\tilde{g}s^{\mu}$-closed set in (X, μ). Since X is $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space, X − {x} is supra α-closed set of (X, μ), that is {x} is supra α-open. (ii) ⇒ (i): Let A be a $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set. Let x ∈ αcl^{μ}(A). By (ii), {x} is either $\tilde{g}s^{\mu}$-closed or supra α-open.

Case (i): If {x} is $\tilde{g}s^{\mu}$-closed. We assume that x ∉ A, then αcl(A) − A contains the $\tilde{g}s^{\mu}$-closed set {x} and A is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed , which cannot happen according to Theorem 4.1. Hence x∈ A.

Case (ii): If {x} is supra α-open, since x ∈ αcl^{μ}(A) then {x} ∩ A = ϕ. This shows that x ∈ A. Thus in both case we have αcl^{μ}(A) ⊂ A. Therefore A = αcl^{μ}(A). That is, A is supra α-closed. Thus X is an $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space.

Definition 4.12 A space (X, μ) is called a $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space if every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set is supra closed.

Theorem 4.13 Every $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space is $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space, but converse not true.

Proof. Since every supra closed set is supra α-closed, the proof follows immediately.

Example 4.14 Let X = {a, b, c}; μ = {∅, {a}, X}. Here (X, μ) is $T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}$-space but not $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space.

Theorem 4.15 If (X, μ) is $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space, then for each x ∈ X, {x} is either $\tilde{g}s^{\mu}$-closed or supra open.

Proof. Suppose (X, μ) is $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space. Let x ∈ X and assume that {x} is not $\tilde{g}s^{\mu}$-closed, then X−{x} is not $\tilde{g}s^{\mu}$-open. Then X is the only $\tilde{g}s^{\mu}$-open set containing X−{x}. Thus X −{x} is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed. Since X is $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space, the set X −{x} is supra closed. Hence {x} is supra open.

Definition 4.16 A space X is called a $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space if every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set in it is $\tilde{g}^{\mu}$-closed.

Theorem 4.17 Every $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space is a $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space but not conversely.

Proof. Let X be a $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space and A be a $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set of X. Since X is a $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space, A is supra-closed. Since every supra-closed set is a $\tilde{g}^{\mu}$-closed. Therefore X is a $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space.

Example 4.18 Let X = {a, b, c, d}, μ = {∅, {a}, {b, c}, {a, b, c}, X}. Here X is an $\tilde{g}^{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space but not a $\#_{T^{\mu}_{\tilde{\tilde{g}}_{\alpha}}}$-space.

5. NEW CLASS OF SUPRA $\tilde{\tilde{g}}_{\alpha}$-CONTINUITY

In this section we introduce and investigate some properties of supra $\tilde{\tilde{g}}_{\alpha}$-continuous functions.

Definition 5.1 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f: (X, τ) → (Y, σ) is called supra $\tilde{\tilde{g}}_{\alpha}$-continuous (briefly $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous) map if the inverse image of every closed set in (Y, σ) is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed in X.

Theorem 5.2 The following statements are true:

(i) Every continuous map is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous.

(ii) Every supra α-continuous map is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous.

(iii) Every $\tilde{g}^{\mu}$-continuous function is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous.

(iv)Every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous function is gs^{μ}-continuous.

(v) Every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous function is sg^{μ}-continuous.

(vi)Every $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous function is $\tilde{\tilde{g}}s^{\mu}$-continuous.

Proof. Here we prove part (vi) only and the remaining parts are trivially follows from Theorem 3.2, 3.5, 3.8, 3.10, and 3.13. Let f : (X, τ) → (Y, σ) be a $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous function and let A be any closed set in Y. Then f^{-1}(A) is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set in (X, μ). Since every$\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed set is $\tilde{\tilde{g}}s^{\mu}$-closed, f^{-1}(A) is $\tilde{\tilde{g}}s^{\mu}$-closed set in (X, μ). Hence f is $\tilde{\tilde{g}}s^{\mu}$-continuous function.

Remark 5.3 The following examples shows that the converse of the above theorem need not be true.

Example 5.4 Consider the set X = {a, b, c}, with the topology τ = {∅, {b, c}, X} and the supra topology μ = {∅, {a}, {b, c}, X}. Suppose Y = {p, q, r}, with the topology σ = {∅, {p}, {p, q}, {p, r}, Y}. Define a map f : (X, τ) → (Y, σ) by f(a) = q, f(b) = r, and f(c) = p. Clearly f is $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous but not supra α-continuous, not supra continuous and not $\tilde{g}^{\mu}$-continuous.

Example 5.5 Consider the sets X = {a, b, c}; Y = {p, q, r} with the topologies τ = {∅, {a}, X} and σ = {∅ , {p}, {p, q}, {p, r}, Y}. The supra topology μ is defined as μ = {∅, {a}, {b}, {a, b}, X}. Define a function f : (X, τ) → (Y, σ) by f(a) = q, f(b) = p, and f(c) = r. Then clearly f is sg^{μ}-continuous and gs^{μ}-continuous but f is not $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous because $f^{-1}(\{q\})$ = {a} is not $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed in (X, μ) where {q} is closed in (Y, σ).

Example 5.6 Let X = {a, b, c, d}; Y = {p, q, r, s} with the topologies τ = {∅, {a}, {a, b, c}, X} and σ = {∅, {p, q, s}, Y}. The supra topology μ is defined by μ = {∅, {a},{b, c}, {a, b, c}, X}. Define a function f : (X, τ) → (Y, σ) by f(a) = s, f(b) = q, f(c) = r and f(d) = s. Then f is $\tilde{\tilde{g}}s^{\mu}$-continuous but f is not $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-continuous because $f^{-1}(\{s\})$ = {a} is not $\tilde{\tilde{g}}_{\alpha}{}^{\mu}$-closed in (X, μ) where {s} is closed in (Y, σ).

Definition 5.7 The supra $\tilde{\tilde{g}}_\alpha$-closure of a set A, denoted by ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A), is the intersection of all ${\tilde{\tilde{g}}_\alpha}^\mu$-closed sets including A. The supra $\tilde{\tilde{g}}_\alpha$-interior of a set A, denoted by ${\tilde{\tilde{g}}_\alpha}^\mu$-int(A), is the union of all supra $\tilde{\tilde{g}}_\alpha$-open sets included A.

Remark 5.8 It is clear that ${\tilde{\tilde{g}}_\alpha}^\mu$-int(A), is a supra $\tilde{\tilde{g}}_\alpha$-open set and ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A), is a supra $\tilde{\tilde{g}}_\alpha$-closed set.

Theorem 5.9

(i) A ⊆ ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A) and A = ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A) if and only if A is a supra $\tilde{\tilde{g}}_\alpha$-closed set.

(ii) ${\tilde{\tilde{g}}_\alpha}^\mu$-int(A) ⊆ A ; and A is a supra $\tilde{\tilde{g}}_\alpha$-open set if and only if A = ${\tilde{\tilde{g}}_\alpha}^\mu$-int(A).

(iii) X − ${\tilde{\tilde{g}}_\alpha}^\mu$-int(A) = ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(X − A)

(iv) X − ${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A) = ${\tilde{\tilde{g}}_\alpha}^\mu$-int(X − A)

Proof. The proof is trivially follows from the definition.

Theorem 5.10 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. Let f be a map from X into Y. Then the following statements are equivalent.

(i) f is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous.

(ii) The inverse image of a closed set in Y is a ${\tilde{\tilde{g}}_\alpha}^\mu$-closed set in X.

(iii) ${\tilde{\tilde{g}}_\alpha}^\mu$-cl($f^{-1}$(A)) ⊆ f^{-1}(cl(A)) for every set A in Y.

(iv) f(${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A)) ⊆ cl(f(A)) for every set A in X.

(v) f(int(B)) ⊆ ${\tilde{\tilde{g}}_\alpha}^\mu$-int(f (B)) for every set B in Y.

Proof. (i) ⇒ (ii): Let A be a closed set in Y, then Y-A is an open set in Y. Then f^{-1}(Y −A) = X−f^{-1}(A) is a ${\tilde{\tilde{g}}_\alpha}^\mu$-open set in X. It follows that f^{-1}(A) is ${\tilde{\tilde{g}}_\alpha}^\mu$-closed subset in X.

(ii) ⇒ (iii): Let A be any subset of Y, since cl(A) is closed set in Y, then f^{-1}(cl(A)) is ${\tilde{\tilde{g}}_\alpha}^\mu$-closed set in X. Therefore ${\tilde{\tilde{g}}_\alpha}^\mu$-cl($f^{-1}$(A)) ⊆ ${\tilde{\tilde{g}}_\alpha}^\mu$-cl($f^{-1}$(cl(A))) = f^{-1}(cl(A)).

(iii) ⇒ (iv): Let A be any subset of X. By (iii), we have f^{-1}(cl(f(A))) ⊇ ${\tilde{\tilde{g}}_\alpha}^\mu$-cl($f^{-1}$(f(A))) =${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A). Hence f(${\tilde{\tilde{g}}_\alpha}^\mu$-cl(A)) ⊆ cl(f(A)).

(iv) ⇒ (v): Let B be any subset in Y. By (iv), f(${\tilde{\tilde{g}}_\alpha}^\mu$-cl(X − f^{-1}(B))) ⊆ cl(f(X −f^{-1}(B))) and f(X −${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B))) ⊆ cl(Y − B) = Y − int(B). Therefore we have X − ${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)) ⊂ f^{-1}(Y − int(B)) and f^{-1}(int(B)) ⊂ ${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)).

(v) ⇒ (i): Let B be any open set in Y and f^{-1}(int(B)) ⊆ ${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)). Then f^{-1}(B) ⊆${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)). But ${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)) ⊆ f^{-1}(B). Hence ${\tilde{\tilde{g}}_\alpha}^\mu$-int($f^{-1}$(B)) = f^{-1}(B). Therefore f^{-1}(B) is ${\tilde{\tilde{g}}_\alpha}^\mu$-open set in X. Thus f is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous function.

Theorem 5.11 Let (X, τ), (Y, σ) and (Z, η) be three topological spaces. If a map f : (X, τ) → (Y, σ) is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous and g : (Y, σ) → (Z, η) is continuous map, then g ∘ f : (X, τ) → (Z, η) is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous.

Proof. Suppose that V be any open set in Z. Since g is continuous, g^{-1}(V) is open in Y.

Since f is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous, f^{-1}(g^{-1}(V)) = (g ∘ f) $^{-1}$(V) is ${\tilde{\tilde{g}}_\alpha}^\mu$-open in X. Therefore g ∘ f is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous.

Theorem 5.12 If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are continuous map, then
g ∘ f : (X, τ) → (Z, η) is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous.

Proof. Since composition of two continuous functions is continuous, and since every continuous function is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous, g ∘ f is ${\tilde{\tilde{g}}_\alpha}^\mu$-continuous.

6. CONCLUSION

In this paper we have introduced supra $\tilde{\tilde{g}}_\alpha$-closed sets in supra topological spaces and established the relationship between this and other existing supra closed sets. We also defined and characterized the properties of some separation axioms using this supra $\tilde{\tilde{g}}_\alpha$-closed sets. Furthermore we investigated the properties of supra $\tilde{\tilde{g}}_\alpha$-continuous functions. We can extend this into many other research fields of General topology such as fuzzy topological spaces, ideal topological spaces and digital topological spaces.

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