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An Inventory Model for Perishable Goods with Partial Backlogging of Shortages

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Abstract— This study illustrates inventory associated with deteriorating items. Nowadays the incident deterioration has a major impact on the preservation of goods in terms of handling inventory. The significant effect of deterioration has been observed on volatile liquids, fish, vegetables, etc. Here a mathematical model is presented incorporating the effect of deterioration. The model has been developed on an infinite time horizon. The shortage is allowed and backlogged partially. We aim to find out lot-size and back-ordered quantities in order to minimize the total average cost. In support of the proposed model, a numerical example has been provided. The stability of the solution of that example has been confirmed by performing a sensitivity analysis of key parameters. A graphical representation of cost function regarding decision variables has been displayed.

Keywords— Inventory; perishable items; shortage; partial backlogging.

I. INTRODUCTION

Preservation of goods is always an important problem in the business world. In real life, deterioration affects badly inventory systems. Generally, fruits, vegetables, fish, etc in many situations deteriorate. Also dairy products, medicines are not free from deterioration. The concern is more for those liquids which are volatile. Generally, inventory means the stock of goods. But due to the deterioration factor goods decays. So the cycle of inventory becomes shorter with respect to the situation when deterioration is not disturbing.

Shortages are very familiar in inventory problems. Due to deterioration, inventory depletes at a faster rate. It enhances the possibility of a shortage of goods. The stock-out situation occurs during a shortage. Now two possibilities may appear on basis of backlogging of shortage amount. They are complete backlogging and partial backlogging. Partial backlogging is more relevant regarding the realistic situation. Generally, during stock-out situations people become impatient. They do not want to wait to get their required commodity. As a consequence of this, they fulfill their demand from other sources. So, a portion of demand is going to be lost which may affect the goodwill of a business. This results in an extra cost in the inventory system. This is familiar as lost sale cost.

Considering all these criteria like inventory, deterioration, the occurrence of shortage, way of backlogging of shortages in a collective way, this inventory model has been proposed.

II. LITERATURE SURVEY

Wee [1] illustrated a model in which items deteriorate according to as Weibull distribution. In that model, quantity discount was also taken into consideration. De and Gowsami [2] formulated an inventory model for deteriorating items under a trade credit situation. Random demand situation was also considered in that model. Sana [3] formulated an inventory model for perishable items under a price-dependent demand situation. In that model, the demand rate was taken to be a decreasing quadratic function of price. Supply chain coordination for deteriorating items was executed by Giri and Chakraborty [4]. In that model production process shifts from an in-control state to an out-control state. Pal [5] examined the inventory model for deteriorating items with a finite life cycle. Shah[6] presented a three-stage integrated inventory model for deteriorating items with two levels of trade credit. Sundarajan et al. [7] allowed partial backlogging of shortages in an inventory system with non-instantaneous deteriorating items. Sharifi et al. [8] developed a model for imperfect and deteriorating items. In that model, a test was conducted along with some inspection errors. Pal et al. [9] analyzed a supply chain with two-phase deterioration under credit financing among two members. Mercier and Castro [10] compared maintenance strategies for two imperfect models whose deterioration follows a gamma distribution. Effects of reference price in deteriorating were considered by Heish and Dye [11]. They also assumed inventory level along with selling price-dependent demand rate. Dyranto et al. [12] considered the effect of carbon emission in a supply chain consists of three layers. In that model carbon emissions from different directions as well as from disposing of deteriorating items were taken into consideration. Probabilistic demand and deterioration functions were incorporated in a three-layer chain by Maihami et al. [13]. The fruitfulness of investment in preservation technology from the end of retailer for non-instantaneous deteriorating items was determined by Bardhan et al. [14]. He et al. [15] derived pricing decisions in a dual-channel supply chain involving deteriorating products.

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III. BASIC ASSUMPTIONS WITH NOTATIONS

- An inventory model based on a single stage is considered.
- The demand rate is constant.
- The time horizon is infinite
- The deterioration rate is constant.
- A shortage occurs and is backlogged partially at a constant rate.
- The cost of lost sales is considered.

Notations	Meaning
α	Demand rate
Q	Lot size
$I_1(t)$	Inventory level at time $t \in [0, T_1]$
$I_2(t)$	Inventory level at time $t \in [T_1, T]$
T	Cycle length
c_b	Backorder cost of a unit
l	Lost sale cost of a unit
h	Holding cost of a unit
k	Set up cost
c_p	Purchase cost
θ	Deterioration rate
B	Backorder level
ATC	Average total cost
γ	Backlogging rate

IV. MODEL DEVELOPMENT

The lot size is Q. Among this, quantity B is used for back-ordering. So model starts with inventory level $Q-B$. Inventory is exhausted due to two reasons: demand and deterioration. Inventory reduces to zero at time $t = T_1$. Then shortage occurs. Shortage partially backlogged at a constant rate γ. The inventory cycle ends at time T. Cost components are set up cost, holding cost, shortage cost, or cost of the backlogging, and lost sale cost. The cost minimization problem is formulated. Decision variables are lot size and backorder level.

V. ASSOCIATED DIFFERENTIAL EQUATIONS

Differential equation governing inventory level for the time interval $[0, T_1]$ is given by

$$\frac{dI_1}{dt} = -\theta I_1 - \alpha$$

Subject to boundary conditions $I(0) = Q - B$ and $I(T_1) = 0$ (1)

The solution of the differential equation using initial condition $I(0) = Q - B$ is given by

$$I(t) = (Q - B)\, e^{-\theta t} - \frac{\alpha}{\theta}\left(1 - e^{-\theta t}\right) \tag{2}$$

Now using the relation $I(T_1) = 0$ we get

$$T_1 = \frac{1}{\theta} \log\left[1 + \frac{(Q-B)\theta}{\alpha}\right] \tag{3}$$

Holding cost for the period $[0, M]$ is given by

$$A_1 = h \int_0^{T_1} I_1(t)dt$$

$$= \frac{h}{\theta^2} \langle \theta(Q - B) - \alpha \log\left[1 + \frac{(Q-B)\theta}{\alpha}\right] \rangle \tag{4}$$

Differential equation governing inventory level for the time interval $[T_1, T]$ is given by

$$\frac{dI_2}{dt} = -\gamma\ \alpha \qquad \text{where } 0 < \gamma < 1 \tag{5}$$

Subject to the condition $I(T_1) = 0$

The solution of the differential equation is given by

$$I_2(t) = \gamma\ \alpha\,(T_1 - t) \tag{6}$$

Back ordering cost for the interval $[T_1, T]$ is given by

$$A_2 = C_b \int_{T_1}^{T} \{-I_2(t)\}dt$$

$$= C_b\ \gamma\ \alpha\ \frac{(T - T_1)^2}{2} \tag{7}$$

Due to the impatient nature of customers, a fraction of demand is lost during the time period $[T_1, T]$. So the corresponding cost or cost of lost sale is given by

$$A_3 = l \int_{T_1}^{T} (1 - \gamma)\ \alpha\, dt$$

$$= l\, \alpha\, (1 - \gamma)(T - T_1) \tag{8}$$

Total cost is composed of five components. They are namely set-up cost, holding cost, back-ordering cost lost sale cost, and purchase cost

$$TC = Total\ cost = k + A_1 + A_2 + A_3 + c_p\, Q \tag{9}$$

The average total cost is given by

$$ATC = \frac{1}{T}\ TC \tag{10}$$

Where T is the cycle length. The expression for cycle length is given below.

$$T = \frac{Q}{(1+\theta)\, d(s)} \tag{11}$$

Our task is to minimize ATC with respect to decision variables Q and B.

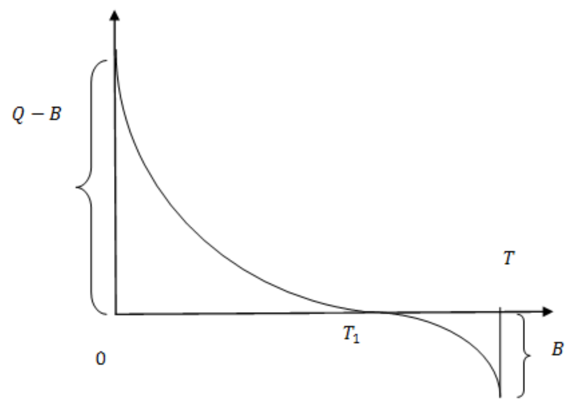

Figure 1: Inventory level (vertical) versus time (horizontal) diagram

VI. SOLUTION PROCEDURE

The necessary condition for maximization is

$$\frac{\partial}{\partial Q}(ATC) = 0, \frac{\partial}{\partial B}(ATC) = 0$$

From these equations, we can obtain optimal values of lot size and backorder level as Q^*, B^*

Sufficient condition for optimality

The Hessian matrix is given below

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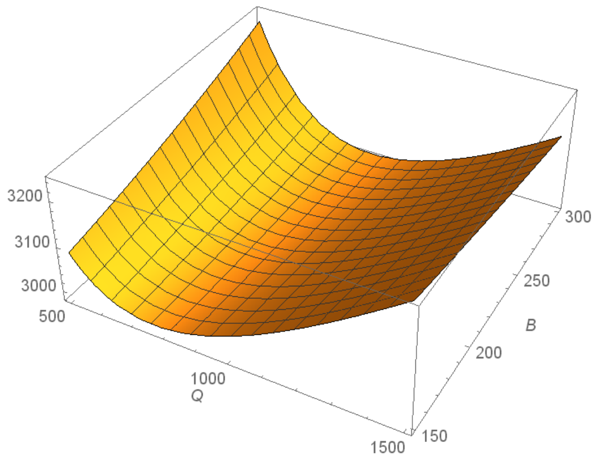
$$\begin{pmatrix} \frac{\partial^2}{\partial Q^2}(ATC) & \frac{\partial^2}{\partial Q\,\partial B}(ATC) \\ \frac{\partial^2}{\partial Q\,\partial B}(ATC) & \frac{\partial^2}{\partial B^2}(ATC) \end{pmatrix}$$

is positive definite at $Q = Q^*, B = B^*$

VII. NUMERICAL EXAMPLE

The following data is used
Backorder cost per unit item $c_b = \$2.0$, Lost sale cost per unit item $l = \$2.5$, Holding cost per unit item per unit time h =\$1.5, Set up cost $k = \$700$, Demand rate $\alpha = 400, \gamma = 0.2, \theta = 0.04$, Purchase cost per unit item $c_p = \$5$
We obtain optimal values of lot size and backorder level $Q^* = 876.65\ units^*and\ B^* = 274.47$ respectively.
Optimal average total cost = \$2983.27. These solutions are optimal since the Eigen values of the Hessian matrix stated above are 0.000206092,00035731. So the Hessian matrix is positive definite.
To solve the problem numerically MATHEMATICA Software has been used.



This is a 3D representation cost function with respect to decision variables $Q\ and\ B$. The graph is concave. So a minimum of cost functions exist certainly. Thus we have proved minimization of cost function not only numerically but also graphically.

VIII SENSITIVITY ANALYSIS

Sensitivity analyses of key parameters are done. The results are shown with the help of a table given below.

Table 1: Variations in Average Total Cost (ATC) and decision variables based on different parameter values.

Parameter	Value of the parameter	Average Total Cost (ATC)	Backorder quantity (B)	Lot Size Q
h	1.3	2936.32	151.16	809.87
	1.4	2961.29	216.76	846.25
	1.5	3002.81	325.72	902.48
	1.6	3020.31	371.59	924.72
k	680	2973.61	249.46	845.20
	690	2978.49	262.08	861.07
	710	2987.98	286.64	891.96
	720	2992.60	298.60	907.01
α	380	2850.21	302.40	885.21
	390	2916.87	288.76	881.34
	410	3049.42	259.51	871.13
	420	3115.31	243.87	864.75
θ	0.02	2940.19	258.48	858.61
	0.03	2961.74	266.43	867.62
	0.05	3004.79	282.50	885.70
	0.06	3026.30	290.56	894.76

- When holding cost h increases average total cost increases. Both back order quantity and lot size increase. Former increases at a faster rate than the latter. The interesting observation is positive inventory level $Q - B$ decreases. So when holding cost is high it is better to keep positive inventory level low.
- With the increase of set up cost (k) average cost increases. There is an increase in the lot size. Large setup cost indicates that the number of set up should be small in number. So a large lot is preferable in this situation.
- As demand rate α increases total average cost increases. Lot size as well as backorder quantity decreases.
- .As deterioration rate θ increase quantity spoils much, so both lot size and backorder quantity should be larger to overcome the event of deterioration.

IX. CONCLUSION

Here an inventory model for deteriorating items under constant demand pattern is formulated. It is a single-stage model. From the numerical example, it can be stated that the model is a cost minimization one. Sensitivity analysis assured as the model is a stable one as there is no abrupt change in quantities as the value of the parameter changes. In this work, the average total cost is minimized by taking ordering lot size and backorder level as decision variables.

X. FUTURE DIRECTIONS OF RESEARCH

This work can be extended by taking a non-constant demand rate. In that case, the demand rate can be assumed as a monotonic decreasing function of the selling price. The transformation from a single stage to two stages is another possible way for the formulation of the new model. Consideration of time-dependent deterioration is another option for the creation of an extension of this work.

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